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Determining the temperature distribution on different boundary conditions on rectangular fin by using shooting method

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Abstract
In this study, the thermal analysis of rectangular fins which thermal conductivity changes by temperature was done. When modelling the fins, finite elements method was used. The effect of different parameters and different boundary conditions to temperature distribution and heat flow was examined for the change of thermal conductivity with temperature and its stability on the rectangular fins. Fin length, width, bottom heat, ambient temperature and heat convection coefficient were searched for thermal analysis. Energy equation was made dimensionless, and finite elements model was formed for the energy equation. Shooting method was used for the numerical solution of energy equation. The results obtained from temperature distribution on rectangular fin in different boundary conditions were compared to the studies in literature.

Keywords: Fin, Thermal Conductivity, Shooting Method

1. Introduction
The fins providing the increase of heat transfer are widely used from micro level to industrial dimensions. The heat and temperature levels transferred are related to the physical characteristics of fin material beside the thermal and geometrical conditions. In most of design studies, the change of thermal conductivity value of fin material by temperature is ignored, but the thermal conductivity of solids changes as a function of temperature. In the fin designs made by ignoring the change of thermal conductivity by temperature, there will be deflections from real values.

In literature, there are a lot of researches considering the change of thermal conductivity by temperature. The solution of ($k = k_a\left[1+\lambda\left(T-T_a\right)\right]$) dimensionless fin energy equation was obtained for temperature dependent temperature conductivity coefficient by using ADM (A domain Decomposition Method). For rectangular fins, temperature distribution along fin and fin yield difference were calculated for λ value [1]. The heating of an infinite-span solid plate at ambient temperature was examined by using the change of thermal conductivity with temperature. Thermal conductivity was taken as $k(T) = 1+\varepsilon T$, and finite elements method was used in the search [2]. Periodical heat transfer on fins was analysed by temperature-dependent heat transmission coefficient and coordinate-dependent heat convection coefficient. Thermal conductivity was taken as the function of $k = k_a\left[1+\lambda\left(T-T_a\right)\right]$ [3]. Thermal conductivity was admitted to change as linear, the approach of integral was used, and then heat transfer from one-dimensional solid surface was researched [4]. Finite volume method was used for determining the temperature distribution of fin. The basic differential equations on thermal conductivity made the heat production with finite element and temperature data linear by using the forms of matrix. The unknown thermal conductivity was solved by using the equations for systems [5] Heat transfer of ring fins was examined by finite difference method and shooting method. The simple numerical solution of generalized Bessel equation was done for hyperbolic ring fins by using both finite difference and shooting methods. The temperature distribution of fin and fin yield were analysed [6]. Fin equation with changeable heat convection coefficient and changeable thermal conductivity was solved by similarity method. Partial differential equation was transformed to ordinary differential equation by choosing an exponential function and by using similarity transformation method. The equation was solved numerically and temperature distribution was obtained for several heat transfer parameters [7].

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In literature, the values of β^* and Ψ as dimensionless thermal conductivity coefficient were chosen regularly. But, all the studies brought the numerical solution to forefront, the values of β^* and Ψ were chosen randomly. Some of the values have no usage in practice. In this study, the values of β^* and Ψ were calculated by considering the practical applications and material characteristics. The value range of β^* and Ψ was decided by taking the results into account.

In many studies, boundary condition of fin whose tip was isolated was generally taken. In this study, 4 different boundary conditions in rectangular fin; 1) fin whose tip was isolated, 2) boundary condition convection, 3) boundary condition for definite temperature and 4) boundary condition of long fin, were examined.

2. Materials and Methods

The geometry of rectangular fin design was shown in Figure (1). When energy conservation was applied to control volume, Equation (1) was obtained. Thermal conductivity states the heat transfer conducted on unit thickness and unit temperature difference, and it changes from material to material. The thermal conductivity of material is dependent on its chemical composition and physical structure. It changes with temperature and pressure.

$$\frac{d}{dx}\left(k(T)A\frac{dT}{dx}\right) - hP\left(T - T_\infty\right) = 0 \qquad (1)$$

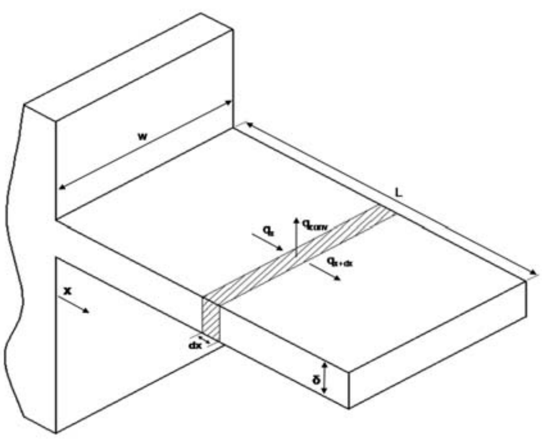


Fig. 1. Thermal conductivity on one dimensional and rectangular fin

The change of thermal conductivity with temperature was stated as linear for many fins in literature especially in small temperature ranges as in Equation (2).

$$k(T) = k_\infty(1+\beta(T-T_\infty)) \qquad (2)$$

Because dimensional thermal analysis was valid for only related geometry and values of boundary condition, in this study dimensionless analysis which is a general approach was preferred with regard to easy to use.

If the value of thermal conductivity in Equation (2) was written in Equation (1) and edited again, Equation (3) was obtained.

$$\left[1+\beta(T-T_\infty)\right]\frac{d^2T}{dx^2} + \beta\left(\frac{dT}{dx}\right)^2 - \frac{hP}{k_\infty A}\left(T-T_\infty\right) = 0 \qquad (3)$$

Dimensionless parameters in Equation (4-8) were used to arrange the fin equation obtained in case thermal conductivity on rectangular fin became the function of temperature.

$$\theta = \frac{T-T_\infty}{T_b - T_\infty}$$: (Dimensionless Temperature) (4)

$$k^*(\theta) = \frac{k(T)}{k_\infty} = 1 + \beta^*\theta$$: (Dimensionless Heat Conduction Coefficient) (5)

$$X^* = \frac{x}{L}$$: (Dimensionless Coordinate)(6)

$$\beta^* = \beta\left(T_b - T_\infty\right)$$: (Multiplier of Conduction Change) (7)

$$\psi = \left(\frac{hPL^2}{k_\infty A}\right)^{1/2}$$: (Thermo-geometric fin parameter) (8)

If dimensionless parameters were written in Equation (1) and it was arranged, then Equation (9) was obtained. Two dimensionless parameters are Ψ and β^* which appear in fin equation and direct the solution of fin equation.

$$\left(1+\beta^*\theta\right)\frac{d^2\theta}{dX^{*2}} + \beta^*\left(\frac{d\theta}{dX^*}\right)^2 - \Psi^2\theta = 0 \qquad (9)$$

β or β^* parameters state the change of thermal conductivity with temperature. If β or β^* parameters are zero, it means that thermal conductivity is accepted as stable. If β or β^* parameters are negative, then it shows that thermal conductivity decreases by temperature and if negative, it means that it increases.

In the studies in literature that the change of thermal conductivity with temperature was considered, the values of β^* were chosen regularly. In the studies, numerical solution was emphasized and values of β^* was chosen randomly. The values of β^* calculated by the way of change of thermal conductivity with temperature were given in Figure (2).

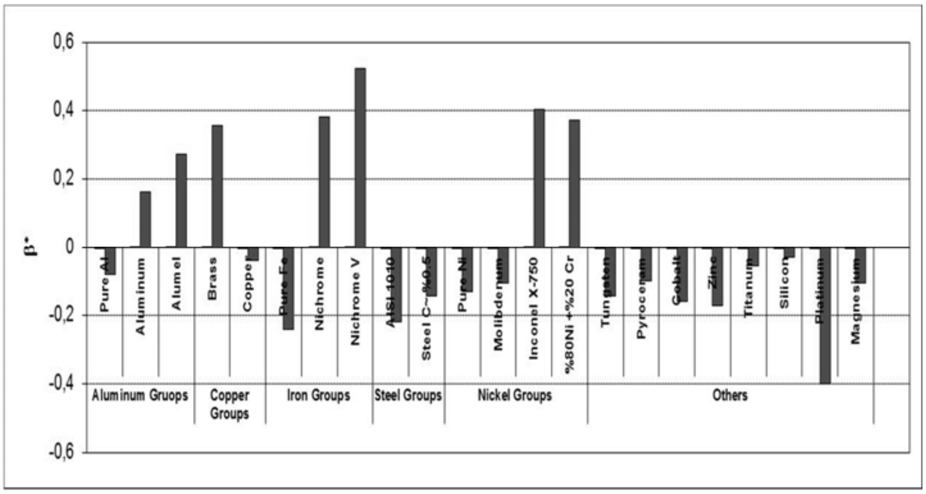


Fig. 2. The values of β* obtained from thermal conductivity of some fin materials used in practical

When Figure 2 was examined, the biggest value of β^* is 0.45. So, in the study values of β^* were accepted in the range of (−0.5) - (0.5) and the analysis was done. When the equation of energy for fin was made dimensionless, dimensionless Ψ appeared in the equation, which is the convection and transmission of geometrical multiplier. The parameters that affect Ψ are the ambient temperature which the fin is in, heat convection coefficient, the length of fin, the thickness of fin and the thermal conductivity related to fin material. Ψ, dimensionless parameter, may also mean the comparison of transmission and convection. The transmitted temperature with the conduction from fin is bigger than the transmitted temperature with convection on the value of Ψ=0.5; the transmitted temperature with the conduction from fin is equal to the transmitted temperature with convection on the value of Ψ=1; the transmitted temperature with the conduction from fin is smaller than the transmitted temperature with convection on the value of Ψ=1.5; the transmitted temperature with the conduction from fin is double bigger than the transmitted temperature with convection on the value of Ψ=2. In the study Ψ, dimensionless parameter, was evaluated on the values of (0.5)-(2) for reviewing all the situations and by considering the results from parametrical studies.

In the study, four boundary conditions were used for obtaining the temperature distributions along the fin. Equation (9) will be solved by applying the boundary conditions to dimensionless fin equation. First boundary condition is the temperature boundary condition identified on fin base for the fin geometry.

$$X^* = 0 \quad \Rightarrow \quad \theta = 1 \tag{10}$$

Temperature distributions along fin will be obtained by applying the four boundary conditions to the fin tip as follows

a) Situation 1: fin whose tip was isolated:

$$X^* = 1 \quad \Rightarrow \quad \frac{d\theta}{dX^*} = 0 \tag{11}$$

b) Situation 2: Boundary condition of convection:

$$X^* = 1 \quad \Rightarrow \quad -k(\theta)\frac{d\theta}{dX^*}\bigg|_{X^*=1} = \frac{hL}{k_\infty}\theta(L) \tag{12}$$

c) Situation 3: Boundary condition of definite temperature:

$$X^* = 1 \quad \Rightarrow \quad \theta = \theta_L = 0.5 \tag{13}$$

d) Situation 4: Boundary condition of long fin:

$$X^* = 1 \quad \Rightarrow \quad \theta = 1 \tag{14}$$

2.1. Shootıng Method

Shooting method is a numeric analysis method solving the boundary value problems by converting them into beginning value. Boundary conditions for solution of Equation (15) used $y(x_0) = y_0$ and $y'(x_1) = y_1$.

$$y''(x) = f(x, y(x), y'(x)) \tag{15}$$

Equation (15) is boundary value function. $y(x_0)=p$, a random value suitable to the boundary condition, which wasn't given related to (x_0) the beginning point of the function can be appointed. A second boundary condition definition in beginning point converts the problem to a beginning value problem. The value of boundary condition of the function in final point must be found for getting hot to the solution of the boundary value problem identified in the beginning. A convergence function as $F(p)=y(x_1;p)-y_1$ is identified. The convergence function taken is used on iteration equation stated in equation (16). The solutions are made with this iterative equation [22].

$$p_n = p_{n-1} - F(p_{n-1})\frac{p_{n-1} - p_{n-2}}{F(p_{n-1}) - F(p_{n-2})} \tag{16}$$

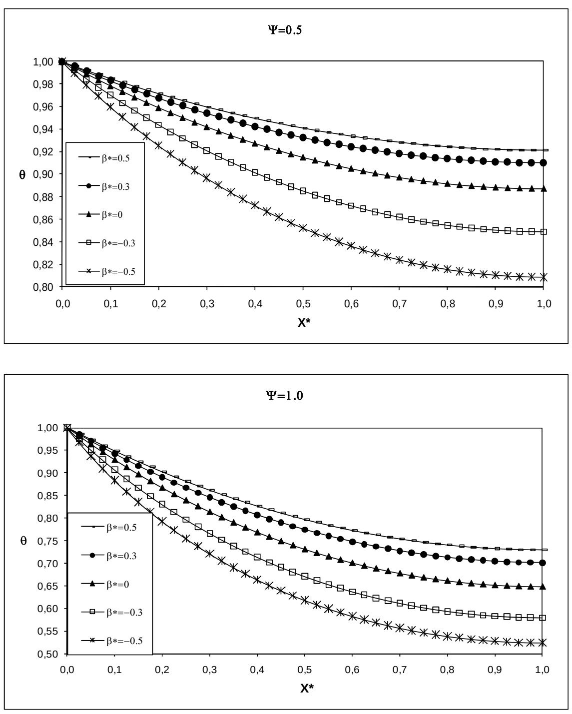
3. Results and Discussion

Energy equation on rectangular fin was made dimensionless and solved by using shooting method for different boundary conditions in case thermal conductivity was the function of temperature. Temperature defined on fin base, isolation on fin tip, convection, temperature and long fin boundary condition were examined on rectangular fin. For stated boundary conditions, dimensionless temperature distribution along the fin and yield difference were obtained for the values of Ψ and β^* in case thermal conductivity coefficient was the function of temperature.

1st Boundary condition: fin whose tip was isolated

In this boundary condition, T_0 temperature was identified on fin base and it is admitted that there was isolation on fin's tip. Temperature distribution along the fin and temperature transition values on fin base were obtained by changing Ψ and β^* in dimensionless temperature equation in definite ranges.

Dimensionless temperature distribution for the values of different Ψ and β^* on fin boundary condition whose tip was isolated was shown in figure (3). The difference among the temperature distributions obtained for the values of different Ψ and β^* in certain boundary condition on rectangular fin is on a level that can't be neglected. The more the value of β^* increased, the more the difference among the temperature distributions increased visibly. The difference between the temperature distribution in case of stable thermal conductivity (β^*=0) and the variable temperature distribution (the other values of β^*) was shown in Figure (3). The increase of the grade of the function means that the thermal conductivity of material changes on a wide range of temperature. It is observed in Figure (3) that when β^* increases, the difference among the temperature distributions increases. When the temperature distributions obtained for β^*=0 and β^*= (-0.5) are compared, it can be clearly seen that there is temperature difference on fin's tip at a rate that cannot be neglected.



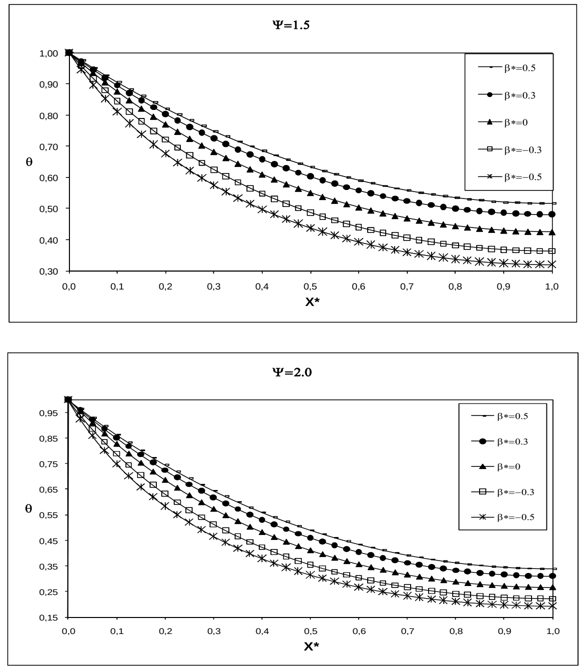
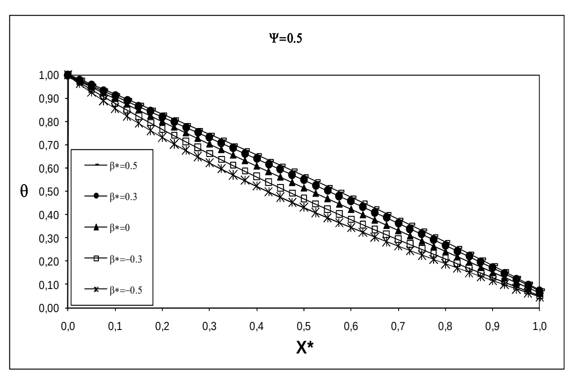


Fig. 3. Dimensionless temperature distribution along the fin for the values of different Ψ and β^* on rectangular fin whose tip is isolated.

2nd Boundary Condition: Convection on fin's tip

It is agreed that there is T_0 temperature identified on fin's base and heat transition by convection on fin's tip. The values of temperature distribution along the fin and heat transition on fin's base were obtained by changing Ψ and β^* in dimensionless temperature equation on stated range. Dimensionless temperature distribution obtained for the values of different Ψ and β^* on heat transition boundary condition and convection on fin's tip was shown in Figure (4). There is a difference among the temperature distributions obtained for the values of different Ψ and β^* or boundary conditions and values on rectangular fin. The more the value of β^* increased, the more the difference among the temperature distributions in the middle of fin increased. The temperature differences on the fin whose tip was isolated are more. Convection boundary condition on fin's tip creates a certain temperature value on the tip point of fin. So, when β^* changes, the temperature value on fin's tip doesn't change like the previous boundary condition.



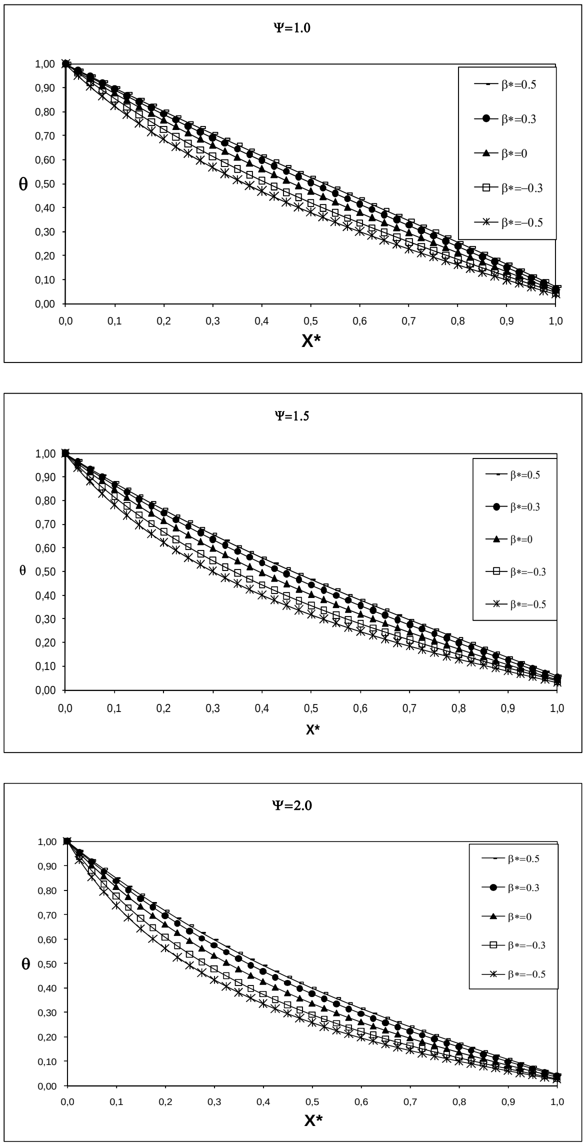


Fig. 4. Dimensionless temperature distribution for values of different Ψ and β^* on the fin whose heat transition boundary condition and convection on rectangular fin's tip were identified.

3rd boundary condition: Definite Temperature on fin's tip

It is agreed that the temperature on fin's base is T_0 and the temperature on fin's tip is T_1. It is admitted that the temperature of the fin's tip is the half of base temperature of the fin. The solution was made as studying dimensionless and admitting that the base temperature of the fin is 1 and the temperature of the fin's tip is 0.2. The values for the temperature distribution along the fin and the heat transition on fin's base were obtained by changing Ψ and β^* in dimensionless temperature equation at stated temperatures. Dimensionless temperature distribution obtained for the values of different Ψ and β^* on temperature boundary conditions defined on the fin's tip was shown in Figure (5). If the temperature distribution values for each β^* on rectangular fin is compared to the temperature values for $\beta^*=0$, it can be clearly observed that the temperature differences are on levels that can't be neglected.

It is shown in Figure (5) that when the value of Ψ increased, the grades of temperature distributions get the logarithmic form. The more the value of Ψ increases, the more the value of heat convection coefficient increases. So, the temperature gradient in the fin increases and the temperature difference on every grade rises. Also, the temperature profile occurred along the fin changes when Ψ increases.

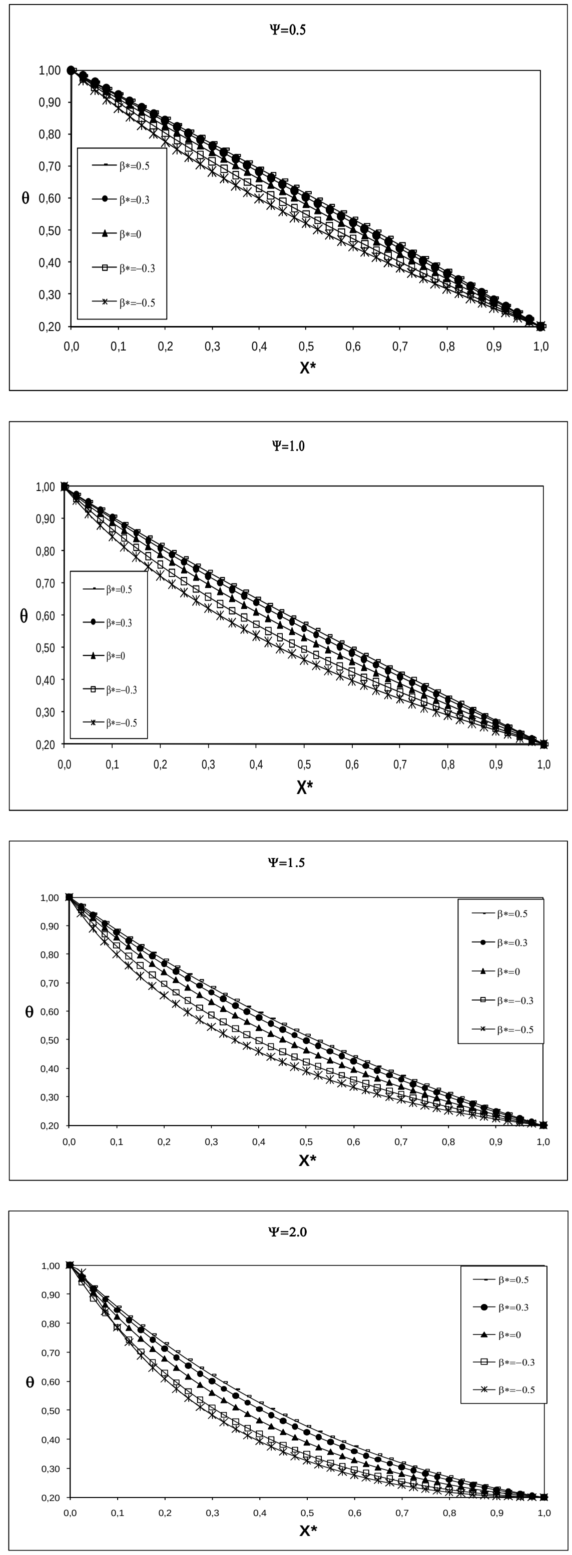
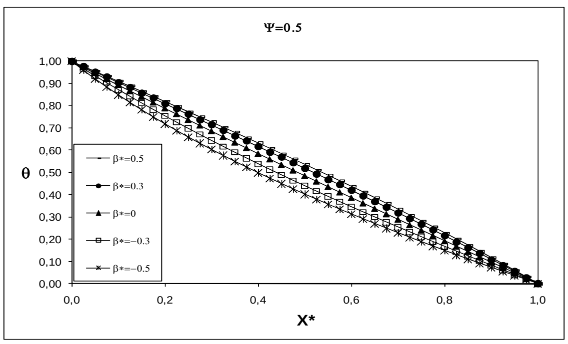
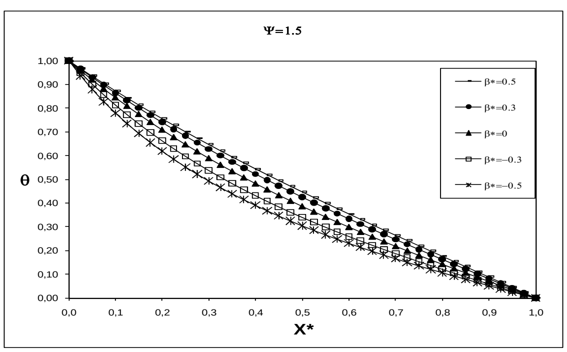


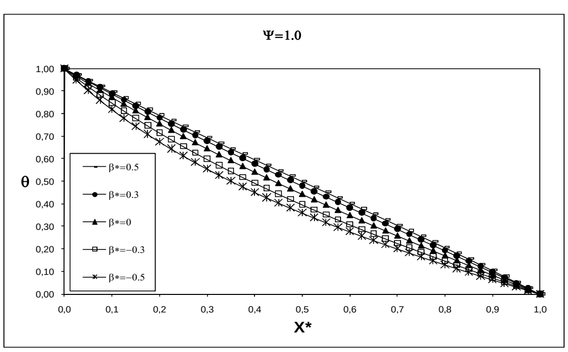
Fig. 5. Dimensionless temperature distribution along the fin for the values of Ψ and β^* different on the fin with the temperature boundary condition defined on rectangular fin's tip.

4th Boundary Condition: Long Fin

The dimensionless thermal analysis of rectangular long fin was done. When the fin length was admitted as very long, the temperature on the fin's tip was the same with the value of ambient temperature where the fin was. It is agreed that the temperature on fin's base is T_0 and the temperature on fin's tip is T_∞ for the heat transition with convection from both top and bottom surface of the fin. The values of temperature distribution and heat transition on the fin's base were obtained for the values of different Ψ and β^* of heat conductivity coefficient. Figure (6) shows dimensionless temperature distribution along the fin for the values of different Ψ and β* on boundary condition of long rectangular fin. When the temperature distributions were examined and Ψ=0.5, it was observed that the difference between the temperature values obtained from β^*=0 and β^*=0.5 analysis in the middle of fin was big. Fin length is more than the other boundary conditions on long fin boundary condition. The surface which was exposed to the heat transition with convection is big. There is more difference between the situations that heat convection coefficient is stable and variable on low Ψ value, which means that the convection is heavy.







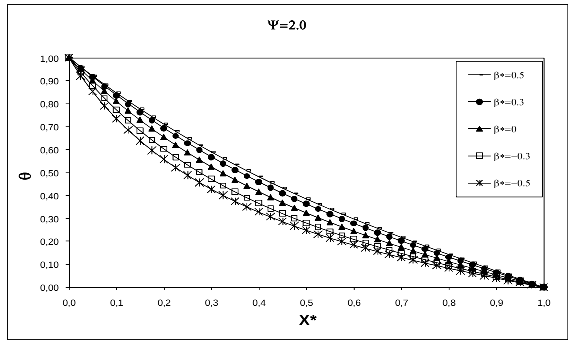
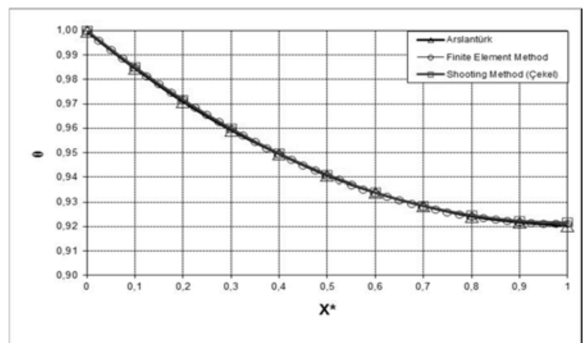
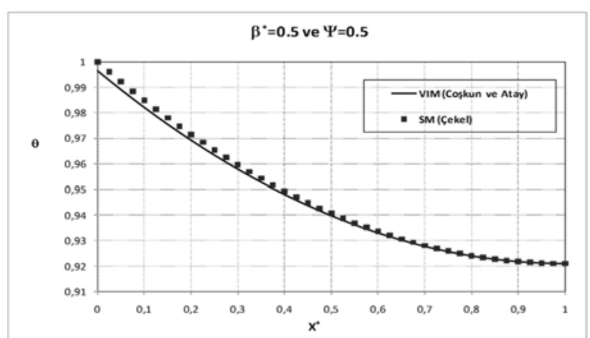


Fig. 6. Dimensionless temperature distribution along the fin for the values of different Ψ and β^* on long and rectangular fin boundary condition.

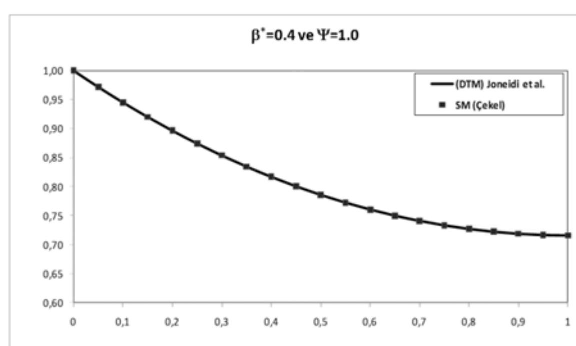
The studies on the fin applications in literature were diversified by using different fin geometries and different solution methods. The results for dimensionless thermal analysis were compared to the values from literature. It was observed that the results of temperature distribution in literature had approximate results to this study.



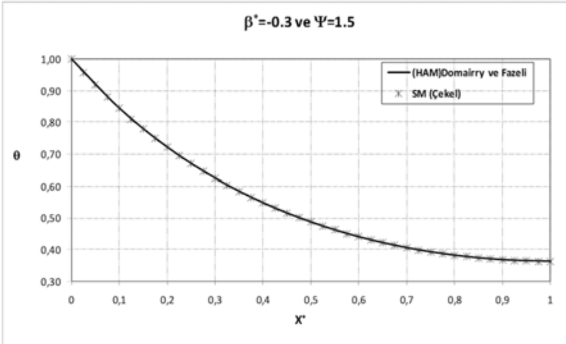
(a) Arslantürk[1]



(b) Coşkun and Atay[21]



(c) Joneidi et al[13]



(d) Domairry and Fazeli[12]

Fig. 7. The comparison of temperature distributions obtained from the literature and Çekel when the heat conduction coefficient is the function of temperature.

4. Conclusions

Temperature and yield distributions for four different boundary conditions on rectangular fin were obtained for the values of different β^* and Ψ. The difference among the temperature distributions of different values of β^* was mostly observed on boundary conditions of fin whose tip was isolated. The more Ψ dimensionless parameter increases on all the boundary conditions, the less the differences among temperature distributions obtained for different β* values decreases. Also, the more Ψ dimensionless parameter increases on all the boundary conditions, the more total heat passing from the fin's base increases. The increase of dimensionless Ψ parameter means increasing of heat convection coefficient. Heat transmission and conduction for high Ψ values lose its significance beside convection. It is confirmed that there is more difference among the temperature distributions obtained for stable and variable thermal conductivity when the transition is heavy, in other saying for low Ψ values. The thermal analysis of thermal conductivity for the values of low Ψ and high β^* must be done by taking the function of temperature.

The thermal analysis for fins working at high temperature ranges must be done by taking the function of temperature for thermal conductivity. When material which has thermal conductivity changing at high temperature ranges was used for fin design, doing thermal analysing considering the thermal conductivity as the function of temperature will increase the yield from the fin. As a result of the thermal analysis, it has been concluded that the thermal analysis done by considering the changing of thermal conductivity by temperature will give more accurate results.

Nomenclature

A Cross-sectional area of the fin (m2)
H Heat transfer coefficient (Wm-2K-1)
K Thermal conductivity of the fin material (Wm-1K-1)
k_∞ Thermal conductivity at the ambient fluid temperature (Wm-1K-1)
k* Dimensionless thermal conductivity
L Fin length (m)
P Fin perimeter (m)
T Temperature (K)
T_∞ Fluid temperature (K)
T_b Temperature at base of the fin (K)
X Distance measured from the fin tip (m)
X* Dimensionless coordinate

Greek symbols

β The slope of the thermal conductivity – temperature curve (K-1)
θ Dimensionless temperature
ψ Thermo-geometric fin parameter
β* Dimensionless parameter describing variation of the thermal conductivity

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