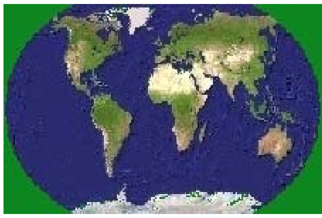

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Two derivative free iterative methods for solving nonlinear equations

Sanjeeda Nazneen

Abstract
This paper proposes two new two-step iterative methods based on forward-difference formula and central-difference formula to eliminate derivative from Kasturiarachi's Leap-frogging Newton's method for solving nonlinear equations. In addition, the order of convergence of the proposed methods also discussed. Moreover, comparisons are taken with some well-known methods.

Keywords: Central-difference formula, forward-difference formula, iterative method, Newton's method, order of convergence.

1. Introduction
Solving the nonlinear equation,

$$f(x) = 0 \tag{1}$$

is one of the fundamental problems in mathematics. In recent years, several methods have been developed to solve the nonlinear equation (1), by using Newton's method, decomposition method, iterative methods, homotopy analysis method, variational iteration method and their modifications [8]. Newton's method is the most widely used iterative method for solving such equations. But it depends on the initial approximation and the closure of initial approximation to actual solution gives quadratically convergent result, otherwise the result diverges. Many authors proposed various iterative methods of higher order convergence in this regard but those methods based on higher order derivatives [5, 9, 8]. In this paper, two two-step iterative methods are constructed by eliminating derivative from Kasturiarcahi's Leap-frogging Newton's method [1].

2. Description of Methods
Kasturiarachi [1] presented Leap-frogging Newton's method of cubic order convergence as

$$x_{n+1} = x_n - \frac{\left(f(x_n)\right)^2}{f'(x_n)\left(f(x_n) - f(\overline{x_n})\right)} \tag{2}$$

where

$$\overline{x_n} = x_n - \frac{f(x_n)}{f'(x_n)} \tag{3}$$

Method 2.1
The forward difference approximation for $f'(x)$ at x is

$$f'(x) \approx \frac{f\left(x + f(x)\right) - f(x)}{f(x)} \tag{4}$$

Replacing x in equation (4), as x_n

$$f'(x_n) \approx \frac{f\left(x_n + f(x_n)\right) - f(x_n)}{f(x_n)} \tag{5}$$

Then substituting the approximated value of $f'(x_n)$ in equation (3) from equation (5),

$$\overline{x_n} = x_n - \frac{f(x_n)}{\left(\dfrac{f\left(x_n + f(x_n)\right) - f(x_n)}{f(x_n)}\right)}$$

i.e.,

$$\overline{x_n} = x_n - \frac{f^2(x_n)}{f\left(x_n + f(x_n)\right) - f(x_n)} \tag{6}$$

Now substituting the value $f'(x_n)$ from equation (5) to equation (2),

$$x_{n+1} = x_n - \frac{\left(f(x_n)\right)^2}{\left(\dfrac{f\left(x_n + f(x_n)\right) - f(x_n)}{f(x_n)}\right)\left(f(x_n) - f(\overline{x_n})\right)}$$

i.e, $x_{n+1} = x_n - \frac{(f(x_n))^3}{(f(x_n+f(x_n))-f(x_n))(f(x_n)-f(\overline{x_n}))}$ (7)

Method 2.2

The Central difference approximation for $f'(x)$ at x is

$$f'(x) \approx \frac{f(x+f(x))-f(x-f(x))}{2f(x)} \quad (8)$$

Replacing x as x_n in equation (8),

$$f'(x_n) \approx \frac{f(x_n+f(x_n))-f(x_n-f(x_n))}{2f(x_n)} \quad (9)$$

Then substituting the approximated value of $f'(x_n)$ from equation (9) to equations (2) and (3) respectively, we get

$$\overline{x_n} = x_n - \frac{2f^2(x_n)}{f(x_n+f(x_n))-f(x_n-f(x_n))} \quad (10)$$

And

$$x_{n+1} = x_n - \frac{2f^3(x_n)}{(f(x_n+f(x_n))-f(x_n-f(x_n)))(f(x_n)-f(\overline{x_n}))} \quad (11)$$

3. Algorithms

Algorithm 3.1:

An approximate solution x_{n+1} can be computed for an initial approximation x_0, by the two step iteration formula

$$x_{n+1} = x_n - \frac{(f(x_n))^3}{(f(x_n+f(x_n))-f(x_n))(f(x_n)-f(\overline{x_n}))} \quad (12)$$

where $\overline{x_n} = x_n - \frac{f^2(x_n)}{f(x_n+f(x_n))-f(x_n)}$ (13)

for $n = 0, 1, 2, \dots$ .....

Algorithm 3.2:

An approximate solution x_{n+1} can be computed for an initial approximation x_0, by the two step iteration formula

$$x_{n+1} = x_n - \frac{2f^3(x_n)}{(f(x_n+f(x_n))-f(x_n-f(x_n)))(f(x_n)-f(\overline{x_n}))} \quad (14)$$

where $\overline{x_n} = x_n - \frac{2f^2(x_n)}{f(x_n+f(x_n))-f(x_n-f(x_n))}$ (15)

for $n = 0, 1, 2, \dots$ .....

4. Convergence Analysis

The proposed method 2.1 has third order convergence, which is discussed in the following theorem 4.1.

Theorem 4.1

Let $a \in I$ be a simple root of sufficiently differentiable function $f: I \subseteq R \to R$ for an open interval I. Then the two step iteration method (Algorithm 3.1) has third order convergence.

Proof:

Let a be a simple root of $f(x) = 0$. So $f(a) = 0$. Since f is sufficiently differentiable. So $f(x_n)$ can be expanded in Taylor's series about $x = a$,

$$f(x_n) = f(a) + f'(a)(x_n - a) + \frac{f''(a)}{2!}(x_n - a)^2 + \frac{f'''(a)}{3!}(x_n - a)^3 + \frac{f^{(4)}(a)}{4!}(x_n - a)^4 + \cdots \dots \dots \dots \quad (16)$$

Let $e_n = x_n - a$ and $\overline{e_n} = \overline{x_n} - a$. Then equation (16) implies,

$$f(x_n) = 0 + f'(a)e_n + \frac{f''(a)}{2!}e_n^2 + \frac{f'''(a)}{3!}e_n^3 + \frac{f^{(4)}(a)}{4!}e_n^4 + O(e_n^5) \quad (17)$$

Squaring $f(x_n)$,

$$f(x_n)^2 = f'(a)^2e_n^2 + f'(a)f''(a)e_n^3 + \left(\frac{1}{4}f''(a)^2 + \frac{1}{3}f'(a)f'''(a)\right)e_n^4 + \cdots \dots \quad (18)$$

Now, $f(x_n + f(x_n)) = f\left(a + e_n + f'(a)e_n + \frac{f''(a)}{2!}e_n^2 + \frac{f'''(a)}{3!}e_n^3 + \frac{f^{(4)}(a)}{4!}e_n^4 + O(e_n^5)\right)$ (19)

Using Taylor series expansion in the right side of equation (19),

$$f(x_n + f(x_n)) = f(a) + f'(a)\left(e_n + f'(a)e_n + \frac{f''(a)}{2!}e_n^2 + \frac{f'''(a)}{3!}e_n^3 + \frac{f^{(4)}(a)}{4!}e_n^4 + O(e_n^5)\right) + \frac{f''(a)}{2!}\left(e_n + f'(a)e_n + \frac{f''(a)}{2!}e_n^2 + \frac{f'''(a)}{3!}e_n^3 + \frac{f^{(4)}(a)}{4!}e_n^4 + O(e_n^5)\right)^2 + \frac{f'''(a)}{3!}\left(e_n + f'(a)e_n + \frac{f''(a)}{2!}e_n^2 + \frac{f'''(a)}{3!}e_n^3 + \frac{f^{(4)}(a)}{4!}e_n^4 + O(e_n^5)\right)^3 + \cdots \dots$$

i.e., $f(x_n + f(x_n)) = (f'(a) + f'(a)^2)e_n + \left(\frac{1}{2}f''(a) + \frac{3}{2}f'(a)f''(a) + \frac{1}{2}f'(a)^2f''(a)\right)e_n^2 + \left(\frac{1}{2}f''(a)^2 + \frac{1}{2}f'(a)f''(a)^2 + \frac{1}{6}f'''(a) + \frac{2}{3}f'(a)f'''(a) + \frac{1}{2}f'(a)^2f'''(a) + \frac{1}{6}f'(a)^3f'''(a)\right)e_n^3 + \left(\frac{1}{8}f''(a)^3 + \frac{5}{12}f''(a)f'''(a) + \frac{2}{3}f'(a)f''(a)f'''(a) + \frac{1}{4}f'(a)^2f''(a)f'''(a) + \frac{1}{24}f'(a)f^{(4)}(a)\right)e_n^4 + O(e_n^5) \dots \dots$

(20)

Then $f(x_n + f(x_n)) - f(x_n) = f'(a)^2e_n + \frac{1}{2}(3 + f'(a))f'(a)f''(a)e_n^2 + \frac{1}{6}(3f''(a)^2 + 3f'(a)f''(a)^2 + 4f'(a)f'''(a) + 3f'(a)^2f'''(a) + f'(a)^3f'''(a)) + \cdots$ ..

(21)

Therefore,

$$\frac{f^2(x_n)}{f(x_n+f(x_n))-f(x_n)} = e_n - \frac{f''(a)}{2f'(a)}(1 + f'(a))e_n^2 + \frac{1}{12f'(a)^2}\left((6 + 6f'(a) + 3f'(a)^2)f''(a)^2 - (4 + 6f'(a) + 2f'(a)^2)f'(a)f'''(a)\right) + \cdots \dots \dots \quad (22)$$

From equation (6), $\overline{x_n} = x_n - \frac{f^2(x_n)}{f(x_n+f(x_n))-f(x_n)}$

i.e., $\overline{x_n} = a + \frac{f''(a)}{2f'(a)}(1 + f'(a))e_n^2 - \frac{1}{12f'(a)^2}\left((6 + 6f'(a) + 3f'(a)^2)f''(a)^2 - (4 + 6f'(a) + 2f'(a)^2)f'(a)f'''(a)\right)e_n^3 + \cdots \dots \dots$ (23)

Hence, $\overline{e_n} = \frac{f''(a)}{2f'(a)}(1 + f'(a))e_n^2 - \frac{1}{12f'(a)^2}\left((6 + 6f'(a) + 3f'(a)^2)f''(a)^2 - (4 + 6f'(a) + 2f'(a)^2)f'(a)f'''(a)\right)e_n^3 + \cdots \dots \dots$ (24)

Therefore, equation (6) has second order convergence.

Now $f(\overline{x_n}) = f\left(a + \frac{f''(a)}{2f'(a)}(1 + f'(a))e_n^2 - \frac{1}{12f'(a)^2}\left((6 + 6f'(a) + 3f'(a)^2)f''(a)^2 - (4 + 6f'(a) + 2f'(a)^2)f'(a)f'''(a)\right)e_n^3 + \cdots \dots \dots\right)$ (25)

Expanding the right side of equation (25) by Taylor series about a,

$$f(\overline{x_n}) = f(a) + f'(a)\left(\frac{f''(a)}{2f'(a)}\left(1+f'(a)\right)e_n^2 - \frac{1}{12f'(a)^2}\left((6+6f'(a)+3f'(a)^2)f''(a)^2 - (4+6f'(a)+2f'(a)^2)f'(a)f'''(a)\right)e_n^3\right) + \frac{f''(a)}{2!}\left(\frac{f''(a)}{2f'(a)}(1+f'(a))e_n^2 - \frac{1}{12f'(a)^2}\left((6+6f'(a)+3f'(a)^2)f''(a)^2 - (4+6f'(a)+2f'(a)^2)f'(a)f'''(a)\right)e_n^3\right)^2 + \cdots \ldots$$

i.e., $$f(\overline{x_n}) = \frac{f''(a)}{2}\left(1+f'(a)\right)e_n^2 - \frac{1}{12f'(a)}\left((6+6f'(a)+3f'(a)^2)f''(a)^2 - (4+6f'(a)+2f'(a)^2)f'(a)f'''(a)\right)e_n^3 + \frac{f''(a)^3\left(1+f'(a)\right)^2}{8f'(a)^2}e_n^4 + \cdots \ldots \ldots \ldots \ldots \quad (26)$$

Hence, $$f(x_n) - f(\overline{x_n}) = f'(a)e_n - \frac{1}{2}f'(a)f''(a)e_n^2 + \left(\frac{f''(a)^2}{12f'(a)}(3f'(a)^2+2f'(a)+6) - \frac{f'''(a)}{6}(f'(a)^2+3f'(a)+1)\right)e_n^3 + \left(-\frac{f''(a)^3}{8f'(a)^2}(1+f'(a))^2 + \frac{1}{24}f^{(4)}(a)\right)e_n^4 + O(e_n^5)$$
(27)

Now, $$f(x_n)^3 = f'(a)^3e_n^3 + \frac{3}{2}f'(a)^2f''(a)e_n^4 + \frac{1}{4}(3f'(a)f''(a)^2 + 2f'(a)^2f'''(a))e_n^4 + O(e_n^5) \quad (28)$$

Taking product of equations (21) and (27),

$$\left(f\left(x_n+f(x_n)\right) - f(x_n)\right)\left(f(x_n) - f(\overline{x_n})\right) = f'(a)^3e_n^2 + \frac{3}{2}f'(a)^2f''(a)e_n^3 + \left(f'(a)f''(a)^2 + \frac{1}{4}f'(a)^2f''(a)^2 + \frac{1}{2}f'(a)^2f'''(a)\right)e_n^4 + O(e_n^5)$$

(29)

Dividing equation (28) by equation (29),

$$\frac{\left(f(x_n)\right)^3}{\left(f(x_n+f(x_n)) - f(x_n)\right)\left(f(x_n) - f(\overline{x_n})\right)} = e_n - \frac{1}{4}\frac{\left(1+f'(a)\right)f''(a)^2}{f'(a)^2}e_n^3 + O(e_n^4) \quad (30)$$

From equations (7) and (30),

$$e_{n+1} = \frac{f''(a)^2}{4f'(a)^2}(1+f'(a))e_n^3 + O(e_n^4)$$
(31)

The following proposed method 2.2 (Algorithm 3.2) has second order convergence which is discussed by the theorem 4.2.

Theorem 4.2

Let $a \in I$ be a simple root of sufficiently differentiable function $f: I \subseteq R \to R$ for an open interval I. Then the two step iteration method (Algorithm 2.2) has second order convergence.

Proof:

For the proof we have equations (16) and (17). Then multiplied equation (18) by 2, we get,

$$2f(x_n)^2 = 2f'(a)^2e_n^2 + 2f'(a)f''(a)e_n^3 + \left(\frac{1}{2}f''(a)^2 + \frac{2}{3}f'(a)f'''(a)\right)e_n^4 + \cdots \ldots \quad (32)$$

We know the value of $f\left(x_n + f(x_n)\right)$ from equation (20).

Then $$f\left(x_n - f(x_n)\right) = (f'(a) - f'(a)^2)e_n + \left(\frac{1}{2}f''(a) - \frac{3}{2}f'(a)f''(a) + \frac{1}{2}f'(a)^2f''(a)\right)e_n^2 + \left(-\frac{1}{2}f''(a)^2 + \frac{1}{2}f'(a)f''(a)^2 + \frac{1}{6}f'''(a) - \frac{2}{3}f'(a)f'''(a) + \frac{1}{2}f'(a)^2f'''(a) - \frac{1}{6}f'(a)^3f'''(a)\right)e_n^3 + \left(\frac{1}{8}f''(a)^3 - \frac{5}{12}f''(a)f'''(a) + \frac{2}{3}f'(a)f''(a)f'''(a) - \frac{1}{4}f'(a)^2f''(a)f'''(a)\right)e_n^4 + O(e_n^5) \ldots \ldots \quad (33)$$

Subtracting equation (33) from equation (20),

$$f\left(x_n + f(x_n)\right) - f\left(x_n - f(x_n)\right) = 2f'(a)^2e_n + 3f'(a)f''(a)e_n^2 + \left(f''(a)^2 + \frac{4}{3}f'(a)f'''(a) + \frac{1}{3}f'(a)^3f'''(a) + O(e_n^4)\right) \quad (34)$$
(34)

Dividing the equation (32) by the equation (34),

$$\frac{2f^2(x_n)}{f\left(x_n+f(x_n)\right) - f\left(x_n-f(x_n)\right)} = e_n - \frac{1}{2}f'(a)f''(a)e_n^2 + \frac{1}{2}f'(a)^2\left(f''(a)^2 - \frac{2}{3}f'(a)f'''(a) - \frac{1}{3}f'(a)^3f'''(a)\right)e_n^3 + O(e_n^4) \quad (35)$$

Substituting equation (35) in equation (10),

$$\overline{x_n} = a + \frac{1}{2}f'(a)f''(a)e_n^2 + f'(a)^2\left(-\frac{1}{2}f''(a)^2 + \frac{1}{3}f'(a)f'''(a) + \frac{1}{6}f'(a)^3f'''(a)\right) + O(e_n^4) \quad (36)$$

To find the value of $f(\overline{x_n})$, we use Taylor's series expansion method,

$$f(\overline{x_n}) = \frac{1}{2}f'(a)^2f''(a)e_n^2 + f'(a)^3\left(-\frac{1}{2}f''(a)^2 + \frac{1}{3}f'(a)f'''(a) + \frac{1}{6}f'(a)^3f'''(a)\right)e_n^3 + O(e_n^4) \quad (37)$$

Subtracting equation (37) from equation (17),

$$f(x_n) - f(\overline{x_n}) = f'(a)e_n + \frac{1}{2}f''(a)(1 - f'(a)^2)e_n^2 + \left(\frac{1}{2}f'(a)^3f''(a)^2 + \frac{1}{6}f'''(a) - \frac{1}{3}f'(a)^4f'''(a) - \frac{1}{6}f'(a)^6f'''(a)\right)e_n^3 + O(e_n^4) \quad (38)$$

Multiplying equations (34) by (38), we get

$$\left(f\left(x_n + f(x_n)\right) - f\left(x_n - f(x_n)\right)\right)\left(f(x_n) - f(\overline{x_n})\right) = 2f'(a)^3e_n^2 + f'(a)^2f''(a)(4 - f'(a)^2)e_n^3 + \left(\frac{5}{2}f'(a)f''(a)^2 - \frac{3}{2}f'(a)^3f''(a)^2 + f'(a)^5f''(a)^2 + \frac{5}{3}f'(a)^2f'''(a) + \frac{1}{3}f'(a)^5f'''(a) - \frac{2}{3}f'(a)^6f'''(a) - \frac{1}{3}f'(a)^8f'''(a)\right)e_n^4 + \left(\frac{1}{2}f''(a)^3 - \frac{1}{2}f'(a)^2f''(a)^3 + \frac{3}{2}f'(a)^4f''(a)^3 + \frac{7}{6}f'(a)f''(a)f'''(a) - \frac{1}{2}f'(a)^3f''(a)f'''(a) - \frac{7}{6}f'(a)^5f''(a)f'''(a) - \frac{1}{2}f'(a)^7f''(a)f'''(a)\right)e_n^5 + O(e_n^6) \quad (39)$$

Multiplying equation (32) by equation (17),

$$2f(x_n)^3 = 2f'(a)^3e_n^3 + 3f'(a)^2f''(a)e_n^4 + \left(\frac{3}{2}f'(a)f''(a)^2 + f'(a)^2f'''(a)\right)e_n^5 + O(e_n^6) \quad (40)$$

Then dividing equation (40) by equation (39),

$$\frac{2f(x_n)^3}{\left(f(x_n+f(x_n))-f(x_n-f(x_n))\right)(f(x_n)-f(\overline{x_n}))}=e_n+\frac{f''(a)}{2f'(a)}(f'(a)^2-1)e_n^2+O(e_n^3) \quad (41)$$

Equation (11) becomes,

$$e_{n+1}=\frac{f''(a)}{2f'(a)}(1-f'(a)^2)e_n^2+O(e_n^3) \quad (42)$$

5. Some well-known methods

5.1 Steffensen's method[3]

This method is a modification of well-known Newton's method based on forward-difference formula given by D. Kincaid and W. Cheney[3] and it is given as

$$x_{n+1}=x_n-\frac{f(x_n)^2}{f(x_n+f(x_n))-f(x_n)} \quad (43)$$

This method is quadratically convergent.

5.2 Chun's method[2]

C. Chun[2] obtained the following iterative method which is cubically convergent:

$$x_{n+1}=x_n-\frac{f(z_{n+1})-f(x_n)}{f'(x_n)} \quad (44)$$

where $$z_{n+1}=x_n+\frac{f(x_n)}{f'(x_n)} \quad (45)$$

5.3 Modified Chun's method with central-difference formula[2]

M. Dehghan and M. Hajarian[6], modified Chun's method by the central-difference approximation to derivative and it is cubically convergent.

$$x_{n+1}=x_n-\frac{2f(x_n)\left(f(z_{n+1})-f(x_n)\right)}{\left(f(x_n+f(x_n))-f(x_n-f(x_n))\right)} \quad (46)$$

where $$z_{n+1}=x_n+\frac{2f(x_n)^2}{\left(f(x_n+f(x_n))-f(x_n-f(x_n))\right)} \quad (47)$$

5.4 Modified Chun's method with forward-difference formula[2]

M. Dehghan and M. Hajarian[6] gave a quadraticially convergent method and they modified Chun's method by approximated the derivative as forward-difference approximation which is given below:

$$x_{n+1}=x_n-\frac{f(x_n)\left(f(z_{n+1})-f(x_n)\right)}{f(x_n+f(x_n))-f(x_n)} \quad (48)$$

where $$z_{n+1}=x_n+\frac{f(x_n)^2}{f(x_n+f(x_n))-f(x_n)} \quad (49)$$

5.5 Modified Newton's method[4]

F. A. Potra and V. Pták[4] made a modification of the well-known Newton's method which has third order convergence and it is given as

$$x_{n+1}=x_n-\frac{f(y_{n+1})+f(x_n)}{f'(x_n)} \quad (50)$$

where $$y_{n+1}=x_n-\frac{f(x_n)}{f'(x_n)} \quad (51)$$

5.6 Modification of modified Newton's method by central-difference formula[4]

M. Dehghan and M. Hajarian[6] substituted the first derivative by the central-difference approximation and they got the following third order convergent method:

$$x_{n+1}=x_n-\frac{2f(x_n)\left(f(y_{n+1})+f(x_n)\right)}{f(x_n+f(x_n))-f(x_n-f(x_n))} \quad (52)$$

where $$y_{n+1}=x_n-\frac{2f(x_n)^2}{f(x_n+f(x_n))-f(x_n-f(x_n))} \quad (53)$$

5.7 Modification of modified Newton's method by forward-difference formula[4]

Also M. Dehghan and M. Hajarian[6] proposed a method which is based on the modified Newton's method[4] and they approximated the first derivative by the forward-difference formula and they found the following second order convergent method:

$$x_{n+1}=x_n-\frac{f(x_n)\left(f(y_{n+1})+f(x_n)\right)}{f(x_n+f(x_n))-f(x_n)} \quad (54)$$

for $$y_{n+1}=x_n-\frac{f(x_n)^2}{f(x_n+f(x_n))-f(x_n)} \quad (55)$$

6. Numerical examples

To make the comparisons and to check the efficiency of the proposed methods, Proposed method 2.1 (PM1), Proposed method 2.2 (PM2), Newton's method with central difference formula (NMCD), we take the other methods such as Newton's method (NM), Steffensen's method (SM), Kasturiarachi's Leap Frogging Newton's method (KLFNM), Chun's method with central difference (CMCD), Chun's method with forward difference (CMFD), Modified Newton's method with central difference (MNMCD), Modified Newton's method with central difference (MNMFD). All iterations are carried out on Mathematica 9.

6.1 Comparisons of methods

To show the comparisons, the following functions are taken from [6, 7].

$f_1(x)=e^x-1.5-\tan^{-1}x$, root = -14.10126977273996

$f_2(x)=\cos x-xe^x+x^2$, root = 0.6391540963320076

Table 6.2: Comparisons of proposed methods with other methods

Function	Initial Approximation	Number of iterations required to find roots								
$f(x)$	x_0	NM	SM	KLFNM	CMCD	CMFD	MNMCD	MNMFD	PM1	PM2
$f_1(x)$	-7	7	7	5	6	5	5	5	5	5
$f_2(x)$	0	7	7	5	6	10	4	4	5	4

From table 6.2, we notice that the proposed methods are more efficient than Newton's method and Steffensen's method and Chun's methods Modified by both central-difference formula and forward-difference formula. But these proposed methods require same number of iterations to get results compared to Kasturiarachi's Leap-frogging method, Modified Newton's methods modified by both central-difference formula and forward-difference formula.

7. Conclusion

In this paper, two one-step iterative methods are constructed based on forward-difference formula and central-difference formula. Also their order of convergences are discussed. Finally, two nonlinear functions are taken to make the comparisons with these proposed methods to some other known methods.

8. References

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