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Creep response in a variable thickness rotating FGM disc

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Abstract

An analytical procedure to investigate steady state creep stresses and strain rates in a linearly variable thickness rotating disc, made of functionally graded material (FGM) is presented. The effects of variation in reinforcement gradation index (m) on the stress and strain rates are investigated. Analysis of steady state creep in a rotating disc made of composites containing SiC_p has been carried out using threshold stress based law with a stress exponent of 5. It is observed that the radial stress exhibits an increase throughout the disc radius; however, the tangential and the effective stresses increase significantly near the inner radius but decrease towards the outer radius with the increase in gradation index. The strain rates in the FGM discs, decreases significantly with the increase in gradation index (m).

Keywords: FGM, Rotating disc, Variable thickness, Steady state creep, Gradation index

Introduction

Rotating disc is a widely used component in engineering and structural applications, involving severe thermo-mechanical loadings where creep becomes significant. Under severe thermo-mechanical loadings, the conventional materials may not survive alone. Thus, a new material concept of Functionally Graded Materials (FGMs) has been developed. In FGMs the constituents or their contents vary with respect to position coordinates, which enable them to offer unique performance [1-3].

Singh and Ray [4] analyzed steady state creep in a rotating isotropic FGM disc of constant thickness by using Norton's power law. The study reveals that the steady state creep response of FGM disc is significantly superior to a similar disc made of Non-FGM composite. Gupta et al [3] extended the work of Singh and Ray [4] to investigate the steady state creep in a constant thickness rotating FGM disc operating under a radial thermal gradient by using sherry's creep law. It is observed that the strain rates in composite disc operating under a radial thermal gradient are lower than those noticed in a similar composite disc, but subjected to a constant average temperature, which is the average of imposed thermal gradient.

Zenkour [5] presented a solution procedure to estimate the distribution of stresses in rotating functionally graded disc of constant thickness. The study indicates that the stresses and deformations in the disc are lower when the material of the disc is tailored in such a way that the elastic modulus and density are more near the inner radius than those at the outer radius. Dwivedi *et al* [6] studied the effect of varying disc thickness profile on creep behavior of rotating FGM disc with linearly varying thickness. The study shows that with the increase in disc thickness gradient, the radial, tangential and effective stresses decrease over the entire disc radius. The strain rates in the disc also reduce significantly with the increase in disc thickness gradient.

The literature consulted reveals that a number of studies have been undertaken to investigate the creep behavior of rotating FGM disc having constant or variable thickness profile. Though, some of the workers investigated creep in variable thickness FGM disc but they assumed linear distribution of reinforcement in the disc. Therefore, it has been decided to investigate the creep behavior of variable thickness FGM disc with different profiles of distribution of reinforcement.

Mathematical Formulation

Considering a Al-SiC_p composite disc of inner and outer radii a ($=31.75$ mm) and b ($=152.4$ mm), respectively. The thickness $h(r)$ of the disc is assumed to vary radially as given below [6],

$$h(r) = h_b + 2k(b - r) \quad (1)$$

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Where, h_a ($= 43.22 \text{ mm}$) and h_b ($=13.97 \text{ mm}$) are disc thickness at the inner and outer radii and k is a constant. The distribution of SiC_p in the FGM disc is assumed to vary with radial distance is given by,

$$V(r) = V_{\max} - \frac{(r-a)^m}{(b-a)^m} (V_{\max} - V_{\min}) = \delta_1 - \delta_2 (r-a)^m \quad (2)$$

where $\delta_1 = V_{\max}$, $\delta_2 = \frac{(V_{\max} - V_{\min})}{(b-a)^m}$, and V_{\max} and V_{\min} are respectively the maximum and minimum SiC_p content at the inner and outer radii respectively. And m is the gradation index.

$$V_{\min} = \frac{3(m^3 + 6m^2 + 11m + 6)V_{av} (b^2 - a^2)t - mV_{\max} \left[3(m+3)(b-a)[(m+1)b + (m+3)a]h_b + 2k(b-a)^2[(m^2 + 6m + 5)b + (2m^2 + 12m + 22)a] \right]}{6[(m+3)(b-a)h_b \{(m+1)b + a\} + 2k(b-a)^2 \{(m+1)b + 2a\}]} \quad (4)$$

The density $\rho(r)$ of the FGM disc at any radius r is estimated by using the rule of mixture as given below,

$$\rho(r) = \frac{[100-V(r)]\rho_m + V(r)\rho_d}{100} = \rho_m + \frac{(\rho_d - \rho_m)V(r)}{100} \quad (5)$$

where r_m ($= 2698.9 \text{ kg/m}^3$) and r_d ($= 3210 \text{ kg/m}^3$) are respectively the densities of pure Al matrix and SiC_p reinforcement [7].

Analysis of creep in FGM disc

The analysis carried out is based on the following assumptions:

- (i) Steady state condition of stress is assumed.
- (ii) Material of the disc is locally isotropic.
- (iii) Elastic deformations in the disc are small and hence neglected as compared to creep deformations.
- (iv) The steady state creep behavior of the disc material is described by the threshold stress based law [8] as,

$$\dot{\varepsilon} = [M(r) \{ \bar{\sigma} - \sigma_0(r) \}]^n \quad (6)$$

The value of true stress exponent (n) in Eq. (6) is taken as 5.

The values of creep parameters $M(r)$ and $\sigma_0(r)$ depend on particle size (P), particle content (V) and operating temperature (T). This dependence is expressed in terms of the following regression equations, developed earlier [9] as,

$$M(r) = 0.0288 - \frac{0.0088}{P} - \frac{14.0267}{T} + \frac{0.0322}{V(r)} \quad (7)$$

$$\sigma_0(r) = -0.084P - 0.023T + 1.185V(r) + 22.207 \quad (8)$$

The effective stress ($\bar{\sigma}$) under biaxial state of stress (*i.e.* $\sigma_z = 0$), as prevails in a rotating disc, may be given by Von-Mises yield criteria as,

$$\bar{\sigma} = \frac{1}{\sqrt{2}} [\sigma_\theta^2 + \sigma_r^2 + (\sigma_r - \sigma_\theta)^2]^{1/2} \quad (9)$$

If V_{av} is the average SiC_p content in the FGM disc and t is the average thickness of the disc, therefore, one may estimate V_{av} from Eq. (2) as,

$$\int_a^b 2\pi r h(r) V(r) dr = V_{av} [\pi(b^2 - a^2)t] \quad (3)$$

Substituting $h(r)$ and $V(r)$ respectively from Eqs. (1) and (2) into Eq. (3), the minimum SiC_p content (V_{\min}) in the FGM disc may be obtained as,

The generalized constitutive equations for creep in an isotropic composite under biaxial state of stress (*i.e.* $\sigma_z = 0$) along the principal directions r , θ and z are [3],

$$\dot{\varepsilon}_r = \frac{d\dot{u}_r}{dr} = \frac{[2x(r)-1]}{2[\{x(r)\}^2 - x(r)+1]^{1/2}} [M(r) \{ \bar{\sigma} - \sigma_0(r) \}]^n \quad (10)$$

$$\dot{\varepsilon}_\theta = \frac{\dot{u}_r}{r} = \frac{[2-x(r)]}{2[\{x(r)\}^2 - x(r)+1]^{1/2}} [M(r) \{ \bar{\sigma} - \sigma_0(r) \}]^n \quad (11)$$

where $x = \sigma_r / \sigma_\theta$ is the ratio of radial and tangential stresses, $\dot{u}_r (= du/dt)$ is the radial deformation rate.

Equations (13) and (14) can be solved to obtain $\sigma_\theta(r)$ as given,

$$\sigma_\theta = \frac{\dot{u}_a^{1/n} \psi^{1/n}(r)}{M(r) [x(r)^2 - x(r)+1]^{1/2}} + \frac{\sigma_0(r)}{[x(r)^2 - x(r)+1]^{1/2}} \quad (12)$$

Multiplying Eq. (12) by $h(r)dr$ and then integrating between limits a to b , we get,

$$\dot{u}_a^{1/n} = \frac{\sigma_{av} \int_a^b h(r) dr - \int_a^b \frac{h(r) \sigma_0(r)}{[x(r)^2 - x(r)+1]^{1/2}} dr}{\int_a^b \frac{h(r) \psi^{1/n}(r)}{M(r)} dr} \quad (13)$$

$$\psi(r) = \frac{2[\{x(r)\}^2 - x(r)+1]^{1/2}}{r[2-x(r)]} \exp\left(\int_a^r \frac{\phi(r)}{r} dr\right) \quad (14)$$

Considering the equilibrium of forces acting on an element of a variable thickness disc, one may write the following force equilibrium equation [10],

$$\frac{d}{dr} [h(r)r\sigma_r] - h(r)\sigma_\theta + \rho(r)\omega^2 r^2 h(r) = 0 \quad (15)$$

Integrating Eq. (18) between limits a to b under the free-free boundary conditions ($\sigma_r = 0$ at $r = a$ and $\sigma_r = 0$ at $r = b$) and using Eqs. (1) and (5), we get,

$$\sigma_{\theta av} = \omega^2 \int_a^b h(r) r^2 \rho(r) dr \bigg/ \int_a^b h(r) dr$$

Knowing the value of average tangential stress $\sigma_{\theta av}$, the distribution of tangential stress in the disc may be estimated by substituting the value of $\dot{u}_a^{1/n}$ from Eq. (13) into Eq. (12).

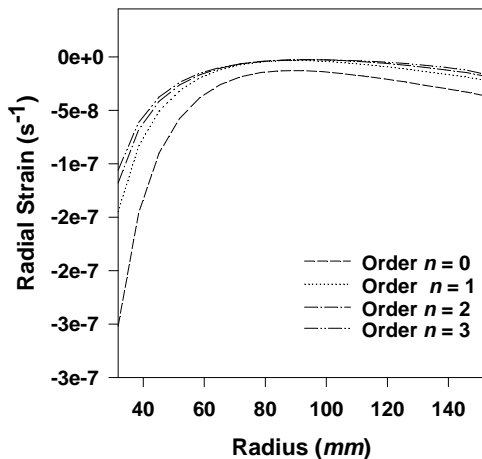
Integrating the equilibrium Eq. (15) between limits a to r , the distribution of radial stress in the disc is obtained as,

$$\sigma_r = \frac{1}{rh(r)} \left[\int_a^r h(r) \sigma_{\theta} dr - \int_a^r h(r) r^2 \rho(r) dr \right] \quad (16)$$

Once the distributions of tangential stress (s_{θ}) and radial stress (s_r) in the disc, are determined by Eqs. (12) and (16), the strain rates $\dot{\epsilon}_r$ and $\dot{\epsilon}_{\theta}$ are estimated respectively from Eqs. (10) and (11).

Results and Discussions

A computer code based on the analysis given in last section has been developed to obtain the distribution of stresses and strain rates in the FGM disc with different reinforcement gradation index. For comparison, the results have also been



obtained for a FGM disc having uniform distribution of SiCp along the radius.

Effect of gradation index (m) on creep behavior of FGM disc

Fig. 1 show the effect of increasing gradation index (m) on creep stresses in the FGM discs. With the increase in gradation index (m) of the FGM disc, the radial stress is observed to increase over the entire disc radii. The increase is higher in the middle of the disc. However, the tangential stress in the disc increases near the inner radius, but decreases towards the outer radius when increasing gradation index (m) increases from 0 to 3.

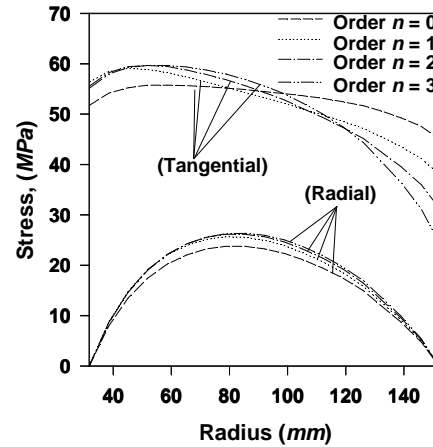


Fig. 1: Variation of radial and tangential stresses in the composite discs.

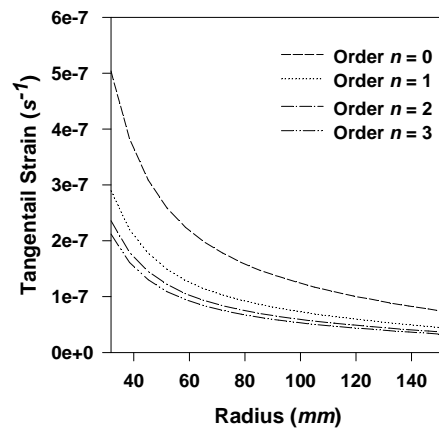


Fig. 2: Effect of reinforcement gradation index (m) on (a) Radial and (b) Tangential strain rate.

The effect of varying gradation index (m) on the strain rates the FGM disc is shown in Figs. 2(a) and 2(b). It is revealed that the radial as well as tangential strain rate in the disc decreases significantly with the increase in gradation index (m) of the disc. The decrease observed is more near the inner radius than that noticed towards the outer radius. It is also evident from Figs. that with increasing gradation index (m) of the disc, the distribution of strain rates in the disc becomes relatively more uniform. Therefore, the FGM disc having respectively higher gradation index (m) will have lesser chances of distortion.

Conclusions

The present study has led to the following conclusions: With the increase in reinforcement gradation index, the radial stress in the FGM disc increases throughout, but the tangential stress increases near the inner radius but decreases towards the outer radius. The decrease observed in tangential stress, towards the outer radius, is slightly higher than the increase noticed near the inner radius. It is also observed that with the increase in reinforcement gradation index, the radial as well as tangential strain rates in the disc decreases throughout. The decrease observed towards the inner radius is slightly higher than that observed towards the outer radius.

References

1. J.F. Durodala and O. Attia, Deformation and stresses in functionally graded rotating disks, *Composite Science and Technology*, 2000; 60: 987-995.
2. M. Bayat, M. Saleem, B.B. Sahari, A.M.S. Hamouda and E. Mahdi, Analysis of functionally graded rotating disks with variable thickness, *Mechanics Research Communications*, 2008; 35: 283-309.
3. V.K. Gupta, S.B. Singh, H.N. Chandrawat and S. Ray, Creep behavior of a rotating functionally graded composite disc operating under thermal gradient, *Metallurgical and Materials Transactions A*, 2004; 35: 1381-1391.
4. S.B. Singh and S. Ray, Steady-state creep behavior in an isotropic functionally graded material rotating disc of Al-SiC composite, *Metallurgical and Materials Transactions A*, 2001; 32: 1679-1685.
5. A.M. Zenkour, Stress distribution in rotating composite structures of functionally graded solid disks, *Journal of Materials Processing Technology*, 2009; 209: 3511-3517.
6. D.D. Dwivedi, V.K. Gupta and A.K. Dham, Investigating the effect of thickness profile of a rotating functionally graded disc on its creep behavior, *Journal of Thermoplastic Composite Materials*, 2013; 26: 461-475.
7. *Metals Handbook* (vol. 2), American Society of Metals, 9th Ed., Metals Park, Ohio, USA, 1978, pp. 714.
8. Z. Y. Ma and S. C. Tjong, Creep deformation characteristics of discontinuously reinforced aluminium matrix composites, *Composites Science and Technology*, 2001; 61(5): 771-786.
9. Manish Garg, B. S. Salaria and V. K. Gupta, Effect of disc geometry on the steady state creep in a rotating disc made of functionally graded materials, *Materials Science Forum*, 2012; 736: 183-191.
10. S.P. Timoshenko, J.N. Goodier, *Theory of Elasticity*, McGraw-Hill, Singapore, 1970.