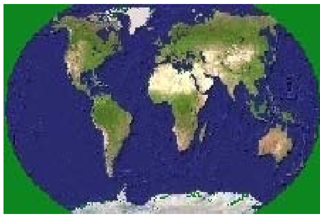

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Creep analysis of a transversely isotropic rotating FGM disc

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Abstract
In this paper, the creep behavior of a transversely isotropic rotating FGM disc is investigated. The SiC_p reinforcement content in the matrix of pure aluminum is non-linearly decreasing along the radius. The creep behavior of the FGM composite disc has been described by threshold stress based law with stress exponent 5. The thickness profile of the rotating disc is linearly decreasing along the radius. It is observed that the radial and tangential stresses changes slightly in FGM disc with α = 1.25 as compare isotropic FGM disc (α = 1). The radial as well as tangential strain rates in the FGM disc are significantly reduce with the increase in extent of anisotropy from 1.0 to 1.25.

Keywords: FGM, Creep, Modeling, Anisotropy, Variable thickness

Introduction

Functionally graded materials (FGMs) are microscopically inhomogeneous composite materials in which the volume fraction of two or more materials is varied smoothly and continuously as a function of position in certain direction(s) of the structure from one point to another (Gupta et al., 2003; Guven et al., 2001). These materials are mainly constructed to operate in high temperature environments and are made from a mixture of metal and ceramic or a combination of different metals. FGMs have been developed as ultra high temperature resistant materials for potential applications in aircrafts, space vehicles and other structural components exposed to elevated temperature (Deepak et al., 2010).

Rotating disc has been the subject of intensive investigations because of its use in different engineering applications (Gupta *et al*., 2005; Garg et al., 2012). Analysis of stresses and strains in solid and annular discs, rotating at high speeds, has been receiving widespread attention due to its wide applications in engineering devices like turbine rotors, flywheels, pumps, disc brakes, etc. (Hojjati and Hassani, 2008, Callioglu *et al*., 2011).

Bhatnagar *et al* (1986) performed creep analysis of an orthotropic rotating disc having different thickness profiles by using Norton's power law. Tutuncu (1995) observed the effect of anisotropy on the stresses and deformations in an orthotropic rotating circular plates. Jain *et al*. (1999) presented an approach to design a constant thickness composite disc of uniform strength by tailoring the anisotropic elastic constants along the radial direction.

Callioglu (2004) investigated stress analysis on a glass-fiber/epoxy orthotropic rotating hollow discs. The temperature distribution is chosen to vary parabolically from inner surface to outer surface along the radial sections. It is observed that when the temperature is increased further, the tangential stress component decreases at the inner surface, whereas increases at the outer surface, and radial stress component are reduced. The radial displacement are higher at the outer surface than at the inner surface. Alexandrova and Alexandrov (2004) investigated the effect of anisotropy on elastic-plastic stresses in a rotating annular disc. To incorporate the effect of anisotropy on the plastic flow, Hill's quadratic orthotropic yield criterion and its associated flow rule were used. The study evaluates the influence of introducing various kinds of anisotropy on stress distribution in a rotating disc.

It is revealed by several investigators that creep strains in variable thickness rotating disc are much lower as compare constant thickness disc. The literature consulted so far reveals that the study pertaining to creep behavior of variable thickness rotating disc made of anisotropic FGM disc is not available. The present study aims to investigate the effect of anisotropy on creep behavior of a rotating FGM disc.

Disc profile and Distribution of Reinforcement

Consider a rotating FGM disc having variable thickness and made of anisotropic Al-SiC_p. The inner and outer radii of the disc are taken respectively as '*a*' and '*b*' and the rpm of the disc is assumed to be 15000. The thickness of the disc is assumed to vary linearly from the inner to outer radius according to the following equation,

$$h(r) = h_b + 2k(b - r) \tag{1}$$

Where, h_a (= 43.22 *mm*) and h_b (=13.97 *mm*) are disc thickness at the inner and outer radii and *k* is a constant.

The content of SiC_p content in the FGM disc is assumed to decrease non-linearly from the inner to outer radius according to the following equation,

$$V(r) = V_{\max} - \frac{(r-a)^2}{(b-a)^2}(V_{\max} - V_{\min}) \tag{2}$$

where V_{max} and V_{min} are respectively the maximum (at inner radius) and the minimum (at outer radius) SiC_p content in the FGM disc.

The average SiC_p content in the FGM disc can be expressed by,

$$V_{av} = \frac{\int_a^b 2\pi r h(r) V(r) dr}{[\pi(b^2 - a^2)t]} \tag{3}$$

Substituting *h*(*r*) and *V*(*r*) respectively from Eqs. (1) and (2) into Eq. (3), we get,

$$V_{\min} = \frac{30V_{av}(b+a)t - V_{\max}[h_b(8b+7a) + h_a(7b+18a)]}{[h_b(12b+3a) + h_a(3b+2a)]} \tag{4}$$

Since density of the disc is a function of SiC_p content, therefore, it will also vary along the radial direction. The density, $\rho(r)$, of the FGM disc at any radius *r* may be estimated from the rule of mixture,

$$\rho(r) = \rho_m + \frac{(\rho_d - \rho_m)V(r)}{100} = A_\rho - B_\rho(r-a)^2 \tag{5}$$

where,

$$A_\rho = \rho_m + (\rho_d - \rho_m)\frac{V_{\max}}{100}$$

and,

$$B_\rho = \frac{(\rho_d - \rho_m)(V_{\max} - V_{\min})}{100(b-a)^2}$$

where ρ_m (= 2698.9 *kg/m*^{3}) and ρ_d (= 3210 *kg/m*^{3}) are respectively the density of pure Al matrix and SiC_p (Clyne and Withers, 1993; Metals Handbook, 1978).

Creep Law

The steady state creep behavior of the composite disc is described by threshold stress (σ_0) based law (Ma and Tjong, 2001) given by,

$$\dot{\bar{\varepsilon}} = [M(r)\{\bar{\sigma} - \sigma_0(r)\}]^n \tag{6}$$

where $M(r)$ and $\sigma_0(r)$ are the creep parameters. The value of true stress exponent (*n*) in Eq. (6) is kept equal to 5.The creep parameters *M*(*r*) and $\sigma_0(r)$ are given by (Deepak *et al.*, 2010),

$$M(r) = 0.0288 - \frac{0.0088}{P} - \frac{14.0267}{T} + \frac{0.0322}{V(r)} \tag{7}$$

$$\sigma_0(r) = -0.084P - 0.023T + 1.185V(r) + 22.207 \tag{8}$$

Analysis of creep in FGM disc

The analysis carried out in this study is based on the following assumptions:

1. The disc is made of transversely isotropic material, *i.e.*, the properties of the disc remain same in the radial (*r*) and axial (*z*) directions but are different in tangential (θ) direction.
2. Elastic deformations in the disc are small and hence neglected as compared to creep deformations.
3. Steady state condition of stress is assumed.
4. The axial stress (σ_z) in the disc remains zero.

The effective stress ($\bar{\sigma}$) in a transversely isotropic rotating disc under biaxial state of stress (*i.e.* σ_z =0) is given by Hill's yield criterion (Dieter, 1988) as,

$$\bar{\sigma} = \frac{1}{\sqrt{G+H}}\left[(G+H)\sigma_r^2 + (H+F)\sigma_\theta^2 - 2H\sigma_r\sigma_\theta\right]^{1/2} = \frac{\left[(2\alpha)\sigma_r^2 + (1+\alpha)\sigma_\theta^2 - 2\alpha\sigma_r\sigma_\theta\right]^{1/2}}{\sqrt{2\alpha}} \tag{9}$$

where, $\alpha = \frac{G}{F} = \frac{H}{F}$ is the coefficient or extent of anisotropy.

The generalized constitutive equations for creep in a transversely isotropic composite under biaxial state of stress takes the following form when the reference frame is along the principal directions r, θ and *z* (Gupta *et al.*, 2005),

$$\dot{\varepsilon}_r = \frac{[(G+H)\sigma_r - H\sigma_\theta]\dot{\bar{\varepsilon}}}{(G+H)\bar{\sigma}} = \frac{[2\sigma_r - \sigma_\theta]\dot{\bar{\varepsilon}}}{2\bar{\sigma}}$$

$$\dot{\varepsilon}_\theta = \frac{[(H+F)\sigma_\theta - H\sigma_r]\dot{\bar{\varepsilon}}}{(G+H)\bar{\sigma}} = \frac{[(1+\alpha)\sigma_\theta - \alpha\sigma_r]\dot{\bar{\varepsilon}}}{(2\alpha)\bar{\sigma}} \tag{10}$$

where, $\dot{\varepsilon}_r, \dot{\varepsilon}_\theta$ and σ_r, σ_θ are respectively the strain rates and the stresses in the disc along *r* and θ directions.

Substituting values of $\dot{\bar{\varepsilon}}$ from Eq. (6) and $\bar{\sigma}$ from Eq. (9) into first equation amongst set of Eqs. (10), we get,

$$\dot{\varepsilon}_r = \frac{d\dot{u}_r}{dr} = \frac{\sqrt{\alpha}\,[2x(r) - 1]\,[M(r)\{\bar{\sigma} - \sigma_0(r)\}]^n}{\sqrt{2}\,[2\alpha x^2(r) - 2\alpha x(r) + (1+\alpha)]^{1/2}} \tag{11}$$

$$\dot{\varepsilon}_\theta = \frac{\dot{u}_r}{r} = \frac{[(1+\alpha) - \alpha x(r)]\,[M(r)\{\bar{\sigma} - \sigma_0(r)\}]^n}{\sqrt{2\alpha}\,[2\alpha x^2(r) - 2\alpha x(r) + (1+\alpha)]^{1/2}} \tag{12}$$

where $x = \sigma_r / \sigma_\theta$ is the ratio of radial and tangential stresses, $\dot{u}_r (= du/dt)$ is the radial deformation rate and u is the radial deformation.

$$\sigma_\theta = \frac{\dot{u}_a^{1/n} \sqrt{2\alpha}\, \psi^{1/n}(r)}{M(r)\left[2\alpha x^2(r) + (1+\alpha) - 2\alpha x(r)\right]^{1/2}} + \frac{\sqrt{2\alpha}\, \sigma_0}{\left[2\alpha x^2(r) + (1+\alpha) - 2\alpha x(r)\right]^{1/2}} \tag{13}$$

and,

$$\psi(r) = \frac{\sqrt{2\alpha}\left[2\alpha x^2(r) - 2\alpha x(r) + (1+\alpha)\right]^{1/2}}{r\left[(1+\alpha) - \alpha x(r)\right]} \exp\left(\int_a^r \frac{\phi(r)}{r} dr\right) \tag{14}$$

Multiplying Eq. (13) by $h(r)dr$ and integrating the resulting equation between limits a to b, we get,

$$\dot{u}_a^{1/n} = \frac{\sigma_{av} \int_a^b h(r)\, dr - \int_a^b \frac{h(r)\sqrt{2\alpha}\, \sigma_0(r)}{[2\alpha x(r)^2 - 2\alpha x(r) + (1+\alpha)]^{1/2}} dr}{\int_a^b \frac{h(r)\psi^{1/n}(r)}{M(r)} dr} \tag{15}$$

Equations (11) and (12) can be solved to obtain as given,

where $\sigma_{\theta av}$ is the average tangential stress in the FGM disc, as given by,

Considering the force equilibrium equation for rotating disc may be written as (Timoshenko and Goodier, 1970),

$$\frac{d}{dr}[h(r) r \sigma_r] - h(r)\sigma_\theta + \rho(r)\omega^2 r^2 h(r) = 0 \tag{16}$$

where $\rho(r)$ is the density of disc at any radius r.

Integrating Eq. (16) between limits a to b, under the free-free B.C.'s ($\sigma_r = 0$ at $r = a$ and $\sigma_r = 0$ at $r = b$) we get,

$$\sigma_{\theta av} = \frac{\omega^2}{\left(\int_a^b h(r)\, dr\right)} \left[A_\rho \left(\int_a^b h(r) r^2 dr \right) - B_\rho \left\{ \begin{array}{l} (h_b + 2kb)\left(\dfrac{6b^5 - 15ab^4 + 10a^2b^3 - a^5}{30}\right) \\ -2k\left(\dfrac{10b^6 - 24ab^5 + 15a^2b^4 - a^6}{60}\right) \end{array} \right\} \right] \tag{17}$$

Knowing $\sigma_{\theta av}$, the tangential stress (σ_θ) in the FGM disc may be obtained from Eq. (13).

Integrating the equilibrium Eq. (16) between limits a to r, the radial stress in the disc is obtained as,

$$\sigma_r = \frac{1}{r\, h(r)} \left[\int_a^r h(r)\sigma_\theta dr - \omega^2 \left[A_\rho \left(\int_a^r h(r) r^2 dr \right) - B_\rho \left\{ \begin{array}{l} (h_b + 2kb)\left(\dfrac{6r^5 - 15ar^4 + 10a^2r^3 - a^5)}{30}\right) \\ -2k\left(\dfrac{10r^6 - 24ar^5 + 15a^2r^4 - a^6}{60}\right) \end{array} \right\} \right] \right] \tag{18}$$

Knowing the distribution of σ_θ and σ_r in the disc, the strain rates $\dot{\varepsilon}_r$ and $\dot{\varepsilon}_\theta$ are estimated respectively from Eqs. (11) and (12).

Results and Discussions

A computer code based on the analysis given in last section has been developed to obtain the distribution of stresses and strain rates in the FGM disc with anisotropic properties. The effect of variation in anisotropy constant (α) has been studied on the creep stresses and strain rates in the disc. The value of α greater than unity means the weakening of the disc in the tangential direction as compare to radial and axial directions. But the value of α =1 implies that the disc material is isotropic.

Distribution of stresses and strain rates

The effect of anisotropy on the creep behavior of a variable thickness FGM disc is shown in Figs. 1(a) and 1(b). The results computed for FGM disc with α =1.25 are compared with the results of isotropic FGM disc (α =1). The radial stress in the disc decreases with the increase in extent of anisotropy from 1 to 1.25. The change observed in radial stress is significant in the middle region of the disc but negligible near the inner and outer radii. The effect of varying α on tangential stress is shown in Figure 4(b). As compared to isotropic FGM disc (α =1), the tangential stress increases near the inner radius but decreases towards the outer radius for FGM disc having α <1. The effect of α on tangential stress is just opposite when it's value is greater than 1 (*i.e.* α >1). The effect of varying α on tangential stress is more near the inner radius than that observed near the outer radius.

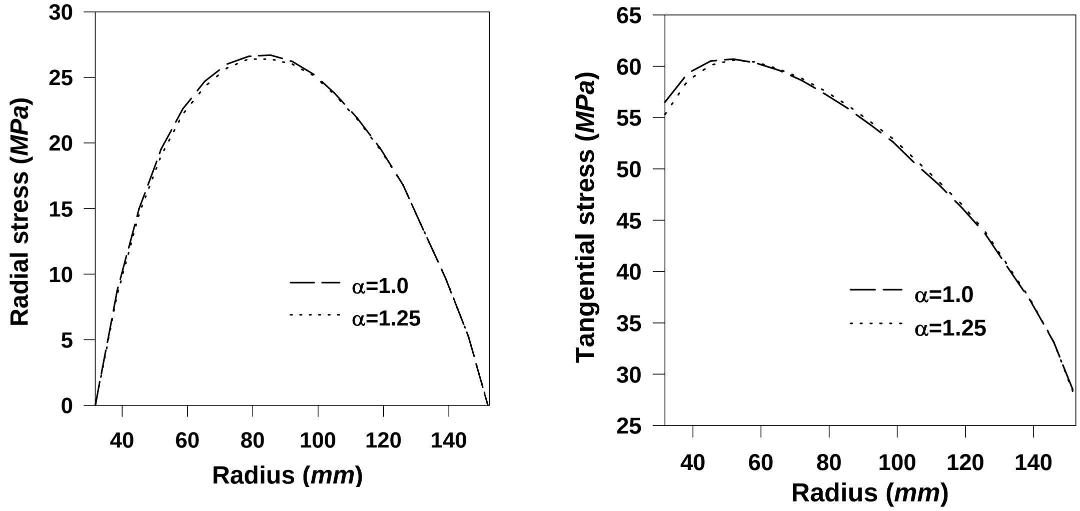

Fig 1: Effect of anisotropy on (a) Radial stress and (b) Tangential stress

As expected the strain rates, radial as well as tangential, decrease significantly over the entire radius, with the increase in α from 0.5 to 1.0 (Figures 6a-6b). The change observed in strain rates is more near the inner radius than that observed towards the outer radius. The FGM disc having α = 1.5 exhibits the lowest strain rates as compared to any other FGM disc. Thus, it is evident that by employing FGM disc with higher strength along the radial and axial directions compared to tangential direction, *i.e.* α > 1, the effective stress and strain rates in the disc are significantly reduced.

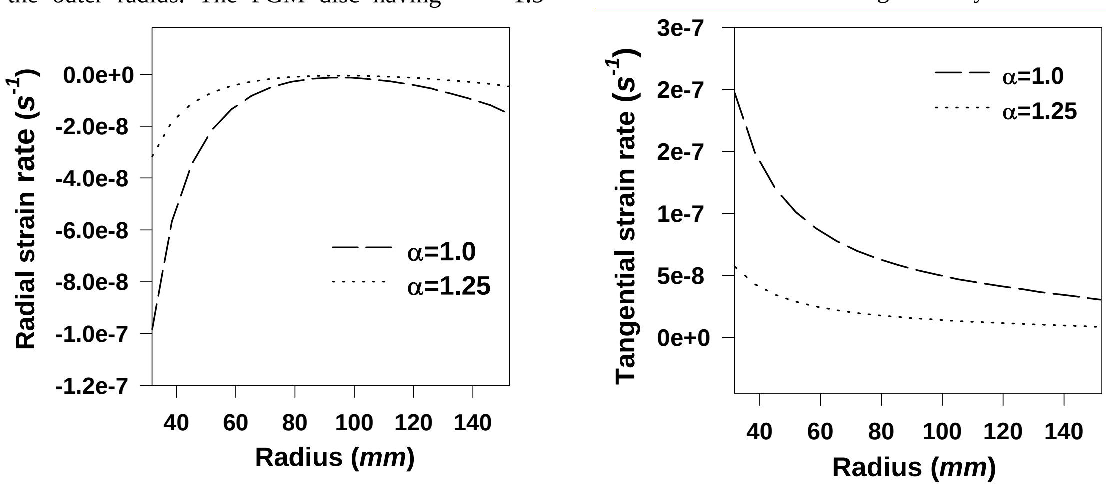

Fig. 2: Effect of anisotropy on (a) Radial strain and (b) Tangential strain rate

Conclusions

The study carried out has led to the following conclusions:

1. The radial as well as tangential stresses in FGM disc are affected in the presence of anisotropy. In anisotropic FGM disc having α >1, the radial stress decreases a little over the entire disc whereas the tangential stress decreases slightly near the inner radius but increases slightly towards the outer radius, when compared with isotropic FGM disc (α =1).
2. The radial and tangential strain rates in the FGM disc reduce significantly with the increase in extent of anisotropy from 1.0 to 1.25.

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