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# Formation of mixed quadrature rules by cleanshaw-curtis five point quadrature rule with another two well-known quadrature rules for numerical integration

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**Abstract**
In this paper, two mixed quadrature rules (I and II) are constructed for approximating the evaluation of numerical integration, by blending Cleanshaw-Curtis five point rule of precision five with another two quadrature rules (five point Fejer's second rule and Boole's rule) of same precision, provided that the new constructed rules are of precision seven. In addition, another mixed quadrature rule (III) has been formed by taking the combination of two newly formed quadrature rules (I and II). Moreover, errors of the constructed rules are analyzed and approximated.

**Keywords:** Cleanshaw-Curtis five point quadrature rule, Five point Fejer's second rule, Boole's rule, Mixed quadrature rule, Numerical integration.

## 1. Introduction

Many Authors formed mixed quadrature rules [1, 12] of higher precision to approximate the real definite integral

$$I(f) = \int_{-1}^{1} f(x)dx \tag{1}$$

by taking the linear combination of two different types of existing quadrature rules of equal lower precision, such as Gauss-Legendre quadrature rules, Newton-Cotes formulas, Fejer's rules, Birkhoff-Young's rule, Boole's rule, Cleanshaw-Curtis rule etc.

Das, R. B. and Debasish D. [5] used mixed quadrature rule by blending Cleanshaw-curtis five point rule [5, 10] and Gauss-Legendre three point rule in adaptive environment. In [5], Dash, R. B. and Debasish D. identified a mixed quadrature rule as Cleanshaw-Curtis five point rule by taken the convex combination of 3-point Fejer's second quadrature rule and Simpson's 1/3 rule of same precision and also identified Cleanshaw-Curtis 7-point rule as the linear combination of 5-point Fejer's second quadrature rule and Boole's rule [5, 8, 9]. In [7], Mohanty, S. K. and Rajani. B. D. formed two mixed quadrature rules, one is the combination of Boole's quadrature rule and Gauss-Legendre three point rule and the other is the combination of Boole's quadrature rule and Bikhoff-Young quadrature rule. Moreover they used the two quadrature rules from [7] to produce another mixed quadrature rule in [8] of precision 9.

In this paper, I intend to construct three mixed quadrature rules by taking the convex combinations of Cleanshaw-Curtis five point quadrature rule$\left(R_{CC5}(f)\right)$with another two quadrature rules, such as five point Fejer's second rule$\left(R_{5F2}(f)\right)$ and Boole's rule $\left(R_{BL}(f)\right)$. In addition, corresponding errors are analyzed and some examples are included to show the accuracy of the approximated value.

## 2. Construction of rules

### 2.1 Construction of mixed quadrature rule (I) of precision 7, by taking convex combination of Cleanshaw-Curtis five point rule and five point Fejer's second rule:

Cleanshaw-Curtis five point rule:

$$I(f) = \int_{-1}^{1} f(x)dx \approx R_{CC5} = \frac{1}{15}\left[f(-1) + 8f\left(-\frac{1}{\sqrt{2}}\right) + 12f(0) + 8f\left(\frac{1}{\sqrt{2}}\right) + f(1)\right] \tag{2}$$

and Five point Fejer's second rule:

$$I(f) = \int_{-1}^{1} f(x)dx \approx R_{5F2} = \frac{2}{45}\left[7f\left(\frac{\sqrt{3}}{2}\right) + 9f\left(\frac{1}{2}\right) + 13f(0) + 9f\left(-\frac{1}{2}\right) + 7f\left(-\frac{\sqrt{3}}{2}\right)\right] \tag{3}$$

Both rules are of precision 5. Let corresponding errors of (3) and (4) are $E_{CC5}(f)$ and $E_{5F2}(f)$ respectively.
For approximating the integral in (1), let $f(x)$ is differentiable in $[-1,1]$.
Then expanding $f(x)$ by Taylor's series about $x=0$,

$$f(x)=f(0)+f'(0)x+\frac{f''(0)}{2!}x^2+\frac{f'''(0)}{3!}x^3+\frac{f^{(4)}(0)}{4!}x^4+\frac{f^{(5)}(0)}{5!}x^5+\frac{f^{(6)}(0)}{6!}x^6+\frac{f^{(7)}(0)}{7!}x^7+\frac{f^{(8)}(0)}{8!}x^8+\frac{f^{(9)}(0)}{9!}x^9+\frac{f^{(10)}(0)}{10!}x^{10}+\cdots\ldots\ldots \quad (4)$$

Let $c_n=\frac{f^{(n)}(0)}{n!}$ for $n=0,1,2,3,\ldots\ldots$ Then (4) becomes,

$$f(x)=c_0+c_1x+c_2x^2+c_3x^3+c_4x^4+c_5x^5+c_6x^6+c_7x^7+c_8x^8+c_9x^9+c_{10}x^{10}+\cdots\ldots\ldots \quad (5)$$

Integrating (5), equation (1) becomes,

$$I(f)=2c_0+\frac{2}{3}c_2+\frac{2}{5}c_4+\frac{2}{7}c_6+\frac{2}{9}c_8+\frac{2}{11}c_{10}+\cdots\ldots \quad (6)$$

As we know, $I(f)=R_{CC5}(f)+E_{CC5}(f)$ (7)
And $I(f)=R_{5F2}(f)+E_{5F2}(f)$ (8)

Approximating errors of (7) and (8), using (6),

$$E_{CC5}(f)=\frac{2}{105}c_6+\frac{1}{45}c_8+\frac{1}{66}c_{10}+\cdots\ldots\ldots \quad (9)$$

and $E_{5F2}(f)=\frac{3}{280}c_6+\frac{1}{45}c_8+\frac{47}{1408}c_{10}+\cdots\ldots\ldots$ (10)

Then multiplying equations (7) and (8) by $\frac{1}{16}$ and $-\frac{1}{9}$ respectively, and then adding the resulting equations,

$$I(f)=\frac{1}{7}\left(16R_{5F2}(f)-9R_{CC5}(f)\right)+\frac{1}{7}\left(16E_{5F2}(f)-9E_{CC5}(f)\right) \quad (11)$$

Equation (12) represents the proposed mixed quadrature rule (I) of convex combination of Cleanshaw-Curtis five point rule and five point Fejer's second rule, $R_{CC5,5F2}(f)$ with error, $E_{CC5,5F2}(f)$.

Then $R_{CC5,5F2}(f)=\frac{1}{7}\left(16R_{5F2}(f)-9R_{CC5}(f)\right)$ (12)

and $E_{CC5,5F2}(f)=\frac{1}{7}\left(16E_{5F2}(f)-9E_{CC5}(f)\right)=\frac{1}{45}c_8+\frac{5}{88}c_{10}+\cdots\ldots\ldots\ldots$ (13)

Hence, $E_{CC5,5F2}(f)=\frac{1}{45}\frac{f^{(8)}(0)}{8!}+\frac{5}{88}\frac{f^{(10)}(0)}{10!}+\cdots\ldots$ (14)

Equation (11) becomes,
$I(f)=R_{CC5,5F2}(f)+E_{CC5,5F2}(f)$ (15)

Therefore, equation (16) represents the proposed rule (I) with the error approximation of precision 7.

**2.2 Construction of mixed quadrature rule (II) of precision 7, by taking convex combination of Cleanshaw-Curtis five point rule and Boole' rule**
Boole's rule:

$$I(f)=\int_{-1}^{1}f(x)dx\approx R_{BL}(f)=\frac{1}{45}\left[7f(-1)+32f\left(-\frac{1}{2}\right)+12f(0)+32f\left(\frac{1}{2}\right)+7f(1)\right] \quad (16)$$

This rule is of precision 5. Let the error of this rule is denoted by $E_{BL}(f)$.

As we know, $I(f)=R_{BL}(f)+E_{BL}(f)$ (17)
From equations (6) and (16),

$$E_{BL}(f)=-\frac{1}{21}c_6-\frac{17}{180}c_8-\frac{23}{176}c_{10}-\cdots\ldots\ldots\ldots \quad (18)$$

Multiplying equations (7) and (17) by $\frac{1}{2}$ and $\frac{1}{5}$ respectively, and then adding,

$$I(f)=\frac{1}{7}\left(5R_{CC5}(f)+2R_{BL}(f)\right)+\frac{1}{7}\left(5E_{CC5}(f)+2E_{BL}(f)\right) \quad (19)$$

Equation (18) represents the proposed mixed quadrature rule (II) of convex combination of Cleanshaw-Curtis five point rule and Boole' rule, $R_{CC5,BL}(f)$ with error, $E_{CC5,BL}(f)$.

Then $R_{CC5,BL}(f)=\frac{1}{7}\left(5R_{CC5}(f)+2R_{BL}(f)\right)$ (20)

And $E_{CC5,BL}(f)=\frac{1}{7}\left(5E_{CC5}(f)+2E_{BL}(f)\right)=-\frac{1}{90}c_8-\frac{7}{264}c_{10}-\cdots\ldots\ldots\ldots$ (21)

Hence, $E_{CC5,BL}(f)=-\frac{1}{90}\frac{f^{(8)}(0)}{8!}-\frac{7}{264}\frac{f^{(10)}(0)}{10!}-\cdots\ldots$ (22)

Therefore, equation (19) becomes,
$I(f)=R_{CC5,BL}(f)+E_{CC5,BL}(f)$ (23)

Therefore, equation (23) represents the proposed rule (I) with the error approximation of precision 7.

**2.3 Construction of mixed quadrature rule (III) of precision 9, by taking linear combinations of rule I and rule II**
From equations (15) and (23),

$$I(f)=R_{CC5,5F2}(f)+E_{CC5,5F2}(f)$$

and $I(f)=R_{CC5,BL}(f)+E_{CC5,BL}(f)$.

with errors $E_{CC5,5F2}(f)=\frac{1}{45}c_8+\frac{5}{88}c_{10}+\cdots\ldots\ldots\ldots\ldots$

and $E_{CC5,BL}(f)=-\frac{1}{90}c_8-\frac{7}{264}c_{10}-\cdots\ldots\ldots\ldots$

Multiplying equation (15) by $\frac{1}{2}$ and then adding with equation (23),

$$I(f)=\frac{1}{3}\left(R_{CC5,5F2}(f)+2R_{CC5,BL}(f)\right)+\frac{1}{3}\left(E_{CC5,5F2}(f)+2E_{CC5,BL}(f)\right) \quad (24)$$

Then equation (24) represents the mixed quadrature rule (III), $R_{(CC5,5F2),(CC5,BL)}(f)$ of linear combinations of mixed quadrature rules (I) and (II) with the approximating error $E_{(CC5,5F2),(CC5,BL)}(f)$.

Then, $R_{(CC5,5F2),(CC5,BL)}(f)=\frac{1}{3}\left(R_{CC5,5F2}(f)+2R_{CC5,BL}(f)\right)$ (25)

And $E_{(CC5,5F2),(CC5,BL)}(f)=\frac{1}{3}\left(E_{CC5,5F2}(f)+2E_{CC5,BL}(f)\right)$ (26)

Hence equation (24) can be written as,
$I(f)=R_{(CC5,5F2),(CC5,BL)}(f)+E_{(CC5,5F2),(CC5,BL)}(f)$ (27)

where error, $E_{(CC5,5F2),(CC5,BL)}(f)=\frac{1}{792}c_{10}+\cdots\ldots\ldots\ldots=\frac{1}{792}\frac{f^{(10)}(0)}{10!}+\cdots\ldots..$ (28)

Then this rule (III) is of precision 9.

## 3. Error Analysis

**Theorem 3.1**

Let $f(x)$ has derivatives of all orders in the interval $[-1,1]$. Then the error, $E_{CC5,5F2}(f)$ associated with the rule, $R_{CC5,5F2}(f)$ is given by

$$\left|E_{CC5,5F2}(f)\right|\approx\frac{1}{45.8!}\left|f^{(8)}(0)\right|.$$

Also $\left|E_{CC5,5F2}(f)\right|\le\frac{M}{29400}|\partial_2-\partial_1|$ ; $\partial_1,\partial_2\in[-1,1]$ for $M=\max_{-1\le x\le 1}\left|f^{(7)}(x)\right|$.

**Proof**

From equation (15),

$$I(f)=R_{CC5,5F2}(f)+E_{CC5,5F2}(f)$$

where $R_{CC5,5F2}(f)=\frac{1}{7}\left(16R_{5F2}(f)-9R_{CC5}(f)\right)$

and $E_{CC5,5F2}(f)=\frac{1}{7}\left(16E_{5F2}(f)-9E_{CC5}(f)\right)$

Then $E_{CC5,5F2}(f)=\frac{1}{45}\frac{f^{(8)}(0)}{8!}+\frac{5}{88}\frac{f^{(10)}(0)}{10!}+\cdots\ldots\ldots\ldots....$

Therefore, $\left|E_{CC5,5F2}(f)\right|\approx\frac{1}{45.8!}\left|f^{(8)}(0)\right|$.

From equations (9) and (10),

$E_{CC5}(f)=\frac{2}{105}c_6+\frac{1}{45}c_8+\frac{1}{66}c_{10}+\cdots\ldots\ldots..=\frac{2}{105}\frac{f^{(6)}(0)}{6!}+\frac{1}{45}\frac{f^{(8)}(0)}{8!}+\frac{1}{66}\frac{f^{(10)}(0)}{10!}+\cdots\ldots\ldots....$ (29)

$E_{5F2}(f)=\frac{3}{280}c_6+\frac{1}{45}c_8+\frac{47}{1408}c_{10}+\cdots\ldots=\frac{3}{280}\frac{f^{(6)}(0)}{6!}+\frac{1}{45}\frac{f^{(8)}(0)}{8!}+\frac{47}{1408}\frac{f^{(10)}(0)}{10!}+\cdots\ldots\ldots..$ (30)

Approximating equations (29) and (30) as,

$E_{CC5}(f)\approx\frac{2}{105}\frac{f^{(6)}(\partial_1)}{6!}=\frac{1}{37800}f^{(6)}(\partial_1)$ (31)

and $E_{5F2}(f)\approx\frac{3}{280}\frac{f^{(6)}(\partial_2)}{6!}=\frac{1}{67200}f^{(6)}(\partial_2)$ (32)

for $\partial_1,\partial_2\in[-1,1]$.

Now $E_{CC5,5F2}(f)=\frac{1}{7}\left(16E_{5F2}(f)-9E_{CC5}(f)\right)$

$$\approx\frac{1}{7}\left(\frac{1}{4200}f^{(6)}(\partial_2)-\frac{1}{4200}f^{(6)}(\partial_1)\right)$$
$$=\frac{1}{29400}\left(f^{(6)}(\partial_2)-f^{(6)}(\partial_1)\right)$$
$$=\frac{1}{29400}\int_{\partial_1}^{\partial_2}f^{(7)}(x)dx,\text{ let }\partial_1<\partial_2.$$

Hence, $\left|E_{CC5,5F2}(f)\right|\approx\left|\frac{1}{29400}\int_{\partial_1}^{\partial_2}f^{(7)}(x)dx\right|\le\frac{1}{29400}\int_{\partial_1}^{\partial_2}\left|f^{(7)}(x)\right|dx=\frac{M}{29400}|\partial_2-\partial_1|$,

where $M=\max_{-1\le x\le 1}\left|f^{(7)}(x)\right|$. It completes the proof.

**Theorem 3.2**

Let $f(x)$ has derivatives of all orders in the interval $[-1,1]$. Then the error, $E_{CC5,5F2}(f)$ associated with the rule, $R_{CC5,5F2}(f)$ is given by

$\left|E_{CC5,BL}(f)\right|\approx\frac{1}{90.8!}\left|f^{(8)}(0)\right|$.

Also $\left|E_{CC5,BL}(f)\right|\le\frac{M}{29400}|\partial_1-\partial_2|$ ; $\partial_1,\partial_2\in[-1,1]$ for $M=\max_{-1\le x\le 1}\left|f^{(7)}(x)\right|$.

**Proof**

From equation (23),

$$I(f)=R_{CC5,BL}(f)+E_{CC5,BL}(f)$$

where $R_{CC5,BL}(f)=\frac{1}{7}\left(5R_{CC5}(f)+2R_{BL}(f)\right)$

and $E_{CC5,BL}(f)=\frac{1}{7}\left(5E_{CC5}(f)+2E_{BL}(f)\right)$

Then $E_{CC5,BL}(f)=-\frac{1}{90}\frac{f^{(8)}(0)}{8!}-\frac{7}{264}\frac{f^{(10)}(0)}{10!}-\cdots\ldots\ldots..$

Approximating $E_{CC5,BL}(f)$, $E_{CC5,BL}(f)\approx-\frac{1}{90}\frac{f^{(8)}(0)}{8!}$

Hence, $\left|E_{CC5,BL}(f)\right|\approx\frac{1}{90.8!}\left|f^{(8)}(0)\right|$.

From equations (29) and (18),

$$E_{CC5}(f)=\frac{2}{105}c_6+\frac{1}{45}c_8+\frac{1}{66}c_{10}+\cdots\ldots\ldots..$$
$$=\frac{2}{105}\frac{f^{(6)}(0)}{6!}+\frac{1}{45}\frac{f^{(8)}(0)}{8!}+\frac{1}{66}\frac{f^{(10)}(0)}{10!}+\cdots\ldots\ldots....$$

$E_{BL}(f)=-\frac{1}{21}c_6-\frac{17}{180}c_8-\frac{23}{176}c_{10}-\cdots\ldots=-\frac{1}{21}\frac{f^{(6)}(0)}{6!}-\frac{17}{180}\frac{f^{(8)}(0)}{8!}-\frac{23}{176}\frac{f^{(10)}(0)}{10!}+\cdots....$ (33)

Approximating equations (29) and (30) as,

$$E_{CC5}(f)\approx\frac{2}{105}\frac{f^{(6)}(\partial_1)}{6!}=\frac{1}{37800}f^{(6)}(\partial_1)$$

$E_{BL}(f)=-\frac{1}{21}\frac{f^{(6)}(\partial_2)}{6!}=-\frac{1}{15120}f^{(6)}(\partial_2)$ (34)

for $\partial_1,\partial_2\in[-1,1]$.

Then $E_{CC5,BL}(f)=\frac{1}{7}\left(5E_{CC5}(f)+2E_{BL}(f)\right)$

$$\approx\frac{1}{7}\left(\frac{1}{7560}f^{(6)}(\partial_1)-\frac{1}{7560}f^{(6)}(\partial_2)\right)$$
$$=\frac{1}{52920}\left(f^{(6)}(\partial_1)-f^{(6)}(\partial_2)\right)$$
$$=\frac{1}{52920}\int_{\partial_2}^{\partial_1}f^{(7)}(x)dx,\text{ let }\partial_1>\partial_2$$

Therefore, $\left|E_{CC5,BL}(f)\right|\approx\left|\frac{1}{52920}\int_{\partial_2}^{\partial_1}f^{(7)}(x)dx\right|\le\frac{1}{52920}\int_{\partial_2}^{\partial_1}\left|f^{(7)}(x)\right|dx=\frac{M}{52920}|\partial_1-\partial_2|$

for $M=\max_{-1\le x\le 1}\left|f^{(7)}(x)\right|$. Hence the proof is complete.

**Theorem 3.3**

Let $f(x)$ has derivatives of all orders in the interval $[-1,1]$. Then the error, $E_{(CC5,5F2),(CC5,BL)}(f)$ associated with the rule, $R_{(CC5,5F2),(CC5,BL)}(f)$ is given by

$\left|E_{(CC5,5F2),(CC5,BL)}(f)\right| \approx \frac{1}{792.10!}\left|f^{(10)}(0)\right|$.

Also $\left|E_{(CC5,5F2),(CC5,BL)}(f)\right| \leq \frac{M}{5443200}\left|\partial_2-\partial_1\right|$ ; $\partial_1,\partial_2 \in [-1,1]$ for $M=\max_{-1\leq x\leq 1}\left|f^{(9)}(x)\right|$.

**Proof**

From equation (27),

$$I(f)=R_{(CC5,5F2),(CC5,BL)}(f)+E_{(CC5,5F2),(CC5,BL)}(f)$$

where $R_{(CC5,5F2),(CC5,BL)}(f)=\frac{1}{3}\left(R_{CC5,5F2}(f)+2R_{CC5,BL}(f)\right)$

and $E_{(CC5,5F2),(CC5,BL)}(f)=\frac{1}{3}\left(E_{CC5,5F2}(f)+2E_{CC5,BL}(f)\right)$.

Then $E_{(CC5,5F2),(CC5,BL)}(f)=\frac{1}{792}c_{10}+\cdots\ldots\ldots\ldots=\frac{1}{792}\frac{f^{(10)}(0)}{10!}+\cdots\ldots..$

Approximating the error, $E_{(CC5,5F2),(CC5,BL)}(f)\approx\frac{1}{792}\frac{f^{(10)}(0)}{10!}$.

Therefore, $\left|E_{(CC5,5F2),(CC5,BL)}(f)\right|\approx\frac{1}{792.10!}\left|f^{(10)}(0)\right|$.

From equations (14) and (22),

$$E_{CC5,5F2}(f)=\frac{1}{45}\frac{f^{(8)}(0)}{8!}+\frac{5}{88}\frac{f^{(10)}(0)}{10!}+\cdots\ldots\ldots\ldots\ldots$$

And $E_{CC5,BL}(f)=-\frac{1}{90}\frac{f^{(8)}(0)}{8!}-\frac{7}{264}\frac{f^{(10)}(0)}{10!}-\cdots\ldots\ldots$

Approximating the above two errors, $E_{CC5,5F2}(f)\approx\frac{1}{45}\frac{f^{(8)}(\partial_2)}{8!}$ and $E_{CC5,BL}(f)\approx-\frac{1}{90}\frac{f^{(8)}(\partial_1)}{8!}$ for $\partial_1,\partial_2\in[-1,1]$.

Then $E_{(CC5,5F2),(CC5,BL)}(f)=\frac{1}{3}\left(E_{CC5,5F2}(f)+2E_{CC5,BL}(f)\right)$

$$\approx\frac{1}{3}\left(\frac{1}{45}\frac{f^{(8)}(\partial_2)}{8!}-\frac{2}{90}\frac{f^{(8)}(\partial_1)}{8!}\right)$$

$$=\frac{1}{5443200}\left(f^{(8)}(\partial_2)-f^{(8)}(\partial_1)\right)$$

$=\frac{1}{5443200}\int_{\partial_1}^{\partial_2}f^{(9)}(x)dx$, Let $\partial_2>\partial_1$.

Hence, $\left|E_{(CC5,5F2),(CC5,BL)}(f)\right|\approx\left|\frac{1}{5443200}\int_{\partial_1}^{\partial_2}f^{(9)}(x)dx\right|\leq\frac{1}{5443200}\left|\int_{\partial_1}^{\partial_2}f^{(9)}(x)dx\right|$

$$=\frac{M}{5443200}\left|\partial_2-\partial_1\right|$$

for $M=\max_{-1\leq x\leq 1}\left|f^{(9)}(x)\right|$.

Therefore, the theorem is proved.

## 4. Results and Discussion

From theorems 3.1, 3.2 and 3.3, we see that

$$\left|E_{(CC5,5F2),(CC5,BL)}(f)\right|\leq\left|E_{CC5,BL}(f)\right|\leq\left|E_{CC5,5F2}(f)\right|$$

From the comparison of errors, we find that mixed quadrature rule III gives more accurate results than quadrature rules I and II.

## 5. Conclusions

Application of quadrature rules are widely used in numerical integration. Here we formed three quadrature rules based on three well known quadrature rules, such as, Cleanshaw-Curtis five point rule, five point Fejer's second rule and Boole's rule and taking combination of them produced more accurately approximated results.

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