International Journal of Multidisciplinary Research and Development

IJMRD 2015; 2(3): 237-239
www.allsubjectjournal.com
Impact factor: 3.672
Received: 28-02-2015
Accepted: 15-03-2015
E-ISSN: 2349-4182
P-ISSN: 2349-5979

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Observation on the hyperbola $y^2=110x^2+1$

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Abstract

The binary quadratic equation $y^2 = 110x^2 + 1$ is considered and a few Properties among the solutions are presented. Employing the intergral solutions of the quadratic equation under consideration, a special Pythagorean triangle is obtained.

Keywords: Binary quadratic, Hyberbola, Integral solutions, Pell equation 2010 Mathematics subject classification: 11D09.

1. Introduction

The binary quadratic equation of the form $y^2 = Dx^2 + 1$, where D is a non-square positive integer, has been studied by various mathematicians for its non-trivial integral solutions when D takes different integral values [1-4]. In [5] infinitely many pythagorean triangle in each of which hypotenuse is four times the product of the generators added with unity are obtained by employing the non-integral solutions of binary quadratic equation of $y^2 = 3x^2 + 1$. In [6], a special pythagorean triangle is obtained by employing the integral solutions of $y^2 = 10x^2 + 1$. In [7], different patterns of infinitely many pythogorean triangle are obtained by employing the non-trivial solutions of $y^2 = 12x^2 + 1$. In this context one may also refer [8-16]. These results have motivated us to search for the integral solutions of yet another binary quadratic equation $y^2 = 110x^2 + 1$ representing a hyperbola. A few interesting properties among the solutions are presented. Employing the integral solutions of the equation under the consideration, a special pythagorean triangle is obtained.

Notations

$t_{m,n}$ = Polygonal number of rank n with size m.

P_n^m = Pyramidal number of rank n with size m.

CP_n^m = Centered pyramidal number of rank n with size m.

$CP_{m,n}$ = Centered polygonal number of rank nwith size m.

GNO_f = Gnomonic number of rank n.

S_f = Star number of rank n.

2. Methods of Analysis

The binary quadratic equation representing hyperbola under consideration is

$$y^2 = 110x^2 + 1 \tag{1}$$

Whose general solution (x_n, y_n) is given by $x_n = \dfrac{g}{2\sqrt{110}}, y_n = \dfrac{f}{2}$,

Where

$$f = \left(21 + 2\sqrt{110}\right)^{n+1} + \left(21 - 2\sqrt{110}\right)^{n+1}$$

g= $(21+2\sqrt{110})^{n+1}-(21-2\sqrt{110})^{n+1}$

, n=0, 1, 2, 3………

The recurrence relations satisfied by x and y are given by

$$y_{n+2}-42y_{n+1}+y_n=0, y_0=21, y_1=881$$

$$x_{n+2}-42x_{n+1}+x_n=0, x_0=2, x_1=84$$

A few numerical examples are given below

n	x_n	y_n
0	2	21
1	84	881
2	3526	36981
3	148008	1552321
4	6212810	65160501
5	260790012	2735188721

From the above table, we observe some interesting properties:

1. x_n is always even.
2. y_n is always odd.
3. $y_{2n}\equiv$ 0 (mod 21).
4. $x_{2n+1}\equiv 1$ (mod 84).
5. $42y_{2n+2}-440x_{2n+2}+2$ is a perfect square.
6. $6(42y_{2n+2}-440x_{2n+2}+2)$ is a nasty number.
7. $42y_{3n+3}-440x_{3n+3}+3(42y_{n+1}-440x_{n+1})$ is a cubic integer.
8. $S_f=6\{42(y_{2n+2}-y_{n+1})-440(x_{2n+2}-x_{n+1})\}$.
9. $GNO_f=84y_{n+1}-880x_{n+1}-1$
10. $y_{n+1}=21y_n-220x_n$
11. $y_{n+2}=881y_n+9240x_n$
12. $x_{n+1}=21x_n+2y_n$
13. $x_{n+2}=881x_n+84y_n$
14. $y_{3n+3}+3y_n=2y_n(y_{2n+1}+1)$
15. $(y_{3n+2}+3y_n)^2=16y_n^6$
16. $21y_{n+2}-881x_{n+1}=220x_n$
17. $21x_{n+1}-2x_{n+1}=x_n$
18. $881x_{n+1}-2y_{n+2}=21x_n$
19. $21x_{n+2}-881x_{n+1}=2y_n$
20. $881y_{n+1}-220x_{n+2}=21y_n$
21. $21y_{n+1}-220x_{n+1}=y_n$
22. $21y_{n+2}-9240x_{n+1}=21y_n$
23. $x_{n+2}-4y_{n+1}=x_n$
24. $881y_{n+2}-84y_{n+2}=x_n$
25. $881y_{n+2}-9240x_{n+2}=y_n$
26. $220x_{n+1}^2-2y_{n+1}^2=x_n y_{n+1}-y_n x_{n+1}$
27. $9240x_{n+2}^2-84y_{n+2}^2=x_n y_{n+2}-y_n x_{n+2}$
28. $21y_{n+2}^2-9240x_{n+2}y_{n+1}=x_n y_{n+2}+y_n y_{n+1}$
29. $2310x_{n+1}^2-21y_{n+1}^2=110x_n x_{n+1}-y_n y_{n+1}$
30. $220x_{n+1}x_{n+2}-2y_{n+1}y_{n+2}=21(x_n y_{n+1}-y_n y_{n+1})$
31. $9240x_{n+1}x_{n+2}-2y_{n+2}^2=21x_n y_{n+2}-y_n x_{n+1}$
32. $21x_{n+2}y_{n+2}-881x_{n+1}y_{n+2}=21y_n x_{n+2}-y_n x_{n+1}$
33. $21x_{n+2}-2y_{n+2}=21x_n+2y_n$
34. $18501y_{n+1}-440y_{n+2}=4620x_n+882y_n$
35. $881x_{n+2}-36960x_{n+1}=x_n+84y_n$
36. $6P_f^m=42\{(m-2)y_{3n+3}+3y_{2n+2}+(2m-1)y_{n+1}\}$ $-440\{(m-2)x_{3n+3}+3x_{2n+2}+(2m-1)x_{n+1}\}+6$
37. $6CP_f^m=42\{m(y_{3n+3}+2y_{n+1})+y_{n+1}\}$ $-440\{(m-2)x_{3n+3}+2x_{n+1}\}+x_{n+1}\}$
38. $2t_{m,f}=42\{(m-2)y_{2n+2}-(m-4)y_{n+1}\}$ $-440\{(m-2)x_{2n+2}-(m-4)_{n+1}\}+2m-4$
39. $2CP_{m,f}=m\{(m-2)(42y_{2n+2}-440x_{2n+2})$ $-(m-2)(42y_{n+1}-440x_{n+1})+2(m-2)\}+2$

Remarkable observation

1. Let $Y = 42y_{n+1} - 440x_{n+1}$ and $X = 21x_{n+1} - 2y_{n+1}$. Then the pair (x, y) satisfied hyperbola $Y^2 = 440X^2 + 4$.
2. Let m, n be two non-zero distinct positive integers such that $m = x_n + 2y_n, n = x_n$

Note that m>n>0. Treat m,n as the generators of the pythogorean triangle T (α, β, γ).

where $\alpha = 2mn$, $\beta = m^2 - n^2$, $\gamma = m^2 + n^2$

Let A, P represent the area and perimeter of T (α, β, γ)
Then the following interesting relations are observed

(a) $\alpha - 55\beta + 54\gamma = -1$

(b) $56\beta - 55\gamma - \frac{4A}{P} = 1$;

(c) $\gamma - 56\alpha + \frac{220A}{P} = 1$

3. Conclusion

To conclude, one may search for other choices of hyperbola for patterns of solutions and their corresponding properties.

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