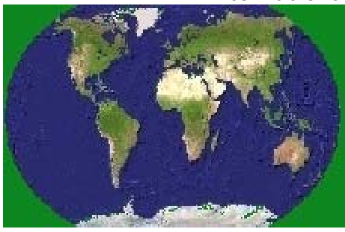
International Journal of Multidisciplinary Research and Development 2015; 2(3): 786-787



International Journal of Multidisciplinary Research and Development

IJMRD 2015; 2(3): 786-787
www.allsubjectjournal.com
Received: 11-03-2015
Accepted: 28-03-2015
e-ISSN: 2349-4182
p-ISSN: 2349-5979
Impact Factor: 3.762

Gopalan.M.A
Professor, Department of Mathematics, Shrimati Indira Gandhi College Trichy-620002, Tamil Nadu, India

Vidhyalakshmi.S
Professor, Department of Mathematics, Shrimati Indira Gandhi College Trichy-620002, Tamil Nadu, India

Lakshmi.K
Lecturer, Department of Mathematics, Shrimati Indira Gandhi College Trichy-620002, Tamil Nadu, India

Correspondence:
Lakshmi.K
Lecturer, Department of Mathematics, Shrimati Indira Gandhi College Trichy-620002, Tamil Nadu, India

On the Non -Homogeneous Sextic Equation $x^4+2(x^2-w)x^2y^2+y^4=z^4$

Gopalan.M.A, Vidhyalakshmi.S, Lakshmi.K

Abstract
We obtain infinitely many non-zero integer quadruples (x, y, z, w) satisfying the non-homogeneous sextic equation with four unknowns $x^4+2(x^2-w)x^2y^2+y^4=z^4$. various interesting properties among the values of x, y, z and w are presented.

Keywords: sextic equation with four unknowns, integral solutions.

1. Introduction

The theory of diophantine equations offers a rich variety of fascinating problems [1-4]. Particularly, in [5, 6], sextic equations with 3 unknowns are studied for their integral solutions. [7, 9] analyse sextic equations with 4 unknowns for their non-zero integer solutions.This communication concerns with an another non-zero sextic equation with 4 unknowns given by $x^4+2(x^2-w)x^2y^2+y^4=z^4$. Infinitely many non-zero integer quadruples (x, y, z, w) satisfying the above equation are obtained. Various interesting properties among the values of x, y, z and w are presented.

2. Method of Analysis:

The diophantine equation representing the sextic equation with four unknowns under consideration is given by

$$x^4+2(x^2-w)x^2y^2+y^4=z^4 \tag{1}$$

(1) Is written as the system of double equations

$$w^2=2y^2+1 \tag{2}$$

$$z^2=wx^2-y^2 \tag{3}$$

(2) Is the well-known pellian equation whose general solution is given by

$$y_n=\frac{1}{2\sqrt{2}}[(3+2\sqrt{2})^{n+1}-(3-2\sqrt{2})^{n+1}] \tag{4}$$

$$w_n=\frac{1}{2}[(3+2\sqrt{2})^{n+1}+(3-2\sqrt{2})^{n+1}] \quad n=0,1,2,3,........ \tag{5}$$

Substitution of (4) & (5) in (3) gives

$$z^2=w_nx^2-y_n^2, \; n=0,1,2,3,........ \tag{6}$$

As it is not possible to obtain a general solution pattern of (6), we have to take particular values of n for getting a general form of solution of (6) corresponding to each value of n.

For illustration, the choice n=1 in (4) and (5) gives $y_1=12, w_1=17$ and thus (6) becomes

$$17x^2-12^2=z^2 \tag{7}$$

Initial solution is $z_0=3, x_0=3$

Consider the pellian $z^2=17x^2+1$ (8)

Initial solution is $\tilde{z}_0=33, \tilde{x}_0=8$ (9)

The general integral solution $(\tilde{x}_n, \tilde{z}_n)$ of (8) is obtained as

$$\tilde{z}_n=\frac{1}{2}[(33+8\sqrt{17})^{n+1}+(33-8\sqrt{17})^{n+1}] \tag{10}$$

$$\tilde{x}_n=\frac{1}{2\sqrt{17}}[(33+8\sqrt{17})^{n+1}-(33-8\sqrt{17})^{n+1}], \; n=0,1,2,3,........ \tag{11}$$

The solution of (7) is obtained as

$$z_{n+1}=z_0\tilde{z}_n+Dx_0\tilde{x}_n=3\tilde{z}_n+51\tilde{x}_n$$

$$x_{n+1}=z_0\tilde{x}_n+x_0\tilde{z}_n=3\tilde{x}_n+3\tilde{z}_n$$

Thus the quadruples $(\tilde{x}_n, 12, \tilde{z}_n, 17)$ represent the non-zero distinct integer solutions of (1).

The recurrence relations among the solutions (x_n, z_n) of (1) are given by

$$x_{n+3} - 66x_{n+2} + x_{n+1} = 0, x_0 = 3, x_1 = 123, x_2 = 8115$$
$$z_{n+3} - 66z_{n+2} - z_{n+1} = 0, z_0 = 3, z_1 = 507, z_2 = 33465$$

2.1 Properties:

1. $17x_{2n+2} - z_{2n+2} + 48$ is a nasty number
2. $9[17x_{3n+3} - z_{3n+3} + 3(17x_{n+1} - z_{n+1})]$ is a cubic integer.
3. $54[17x_{4n+4} - z_{4n+4} + 4(17x_{2n+2} - z_{2n+2}) + 144]$ is a biquadratic integer
4. $17(x_{n+1} - z_{n+1})^2 - 24(17x_{2n+2} - z_{2n+2}) + 1152 = 0$
5. $17x_{2n+2} - z_{2n+2} - 17(z_{n+1} - x_{n+1})^2 = 2$ (triangular number of rank 47)
7. Define: $X = 17x_{2n+2} - z_{2n+2} + 48,\ Y = 17x_{n+1} - z_{n+1}$

It is to be noted that the pair (X, Y) satisfies the parabola $Y^2 = 24X$

3. Conclusion:

In conclusion, by taking other values of n in (6) and following a similar analysis, infinitely many integer quadruples satisfying (1) are determined

4. References:

1. L.E.Dickson, History of Theory of Numbers, Vol.11, Chelsea Publishing Company, New York (1952).
2. L.J.Mordell, Diophantine equations, Academic Press, London(1969).
3. Telang,S.G.,Number theory, Tata Mc Graw Hill publishing company, New Delhi (1996)
4. Carmichael ,R.D.,The theory of numbers and Diophantine Analysis,Dover Publications,New York (1959)
5. M.A.Gopalana and sangeetha, On the sextic equations with 3 unknowns $x^2 - xy + y^2(R^2 + 3)^n z^6$, Impact J.Sci.tech. (2010),Vol 4 No 4,89-93.
6. M.A.Gopalan Manju Somnath and N.Vanitha, Parametrc Solutions of $x^2 - y^6 = z^2$,Acta ciencia indica, (2007),XXXIII,3,1083-1085.
7. M.A.Gopalan and A.Vijaya Sankar, Integral Solutions of the sextic equation $x^4 + y^4 + z^4 = 2w^6$,Indian Journal of Mathematical Sciences, ,(2010),Vol 6,No 2,241-245.
8. \M.A.Gopalan and A.VijayaSankar, Integral Solutions of the sextic equation $x^4 + y^4 + z^4 = 2w^6$,Indian Journal of Mathematics and Mathematical Sciences, (2010), Vol 6,No 2,241-245.
9. M.A.Gopalan,S.Vidhyalakshmi and A.VijayaSankar, Integral Solutions of non-homogeneous sextic equation $xy + z^2 = w^6$, Impact J.Sci.tech,2010, vol 6,No:1, 47-52.