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K. Radha

P.G & Research, Department
of Mathematics, Periyar
E.V.R. College
(Autonomous),
Tiruchirapalli-620023

S. Arumugam

B.T.Asst, Govt. Hr.Sec.
School, Kolakkudi,
Musiri Taluk, Tiruchirapalli-
621208

Double Vertex Graph and Complete Double Vertex Graph of a Fuzzy Graph

K. Radha, S. Arumugam

Abstract

In this paper, we have introduced the concept of double vertex graph and complete double vertex graph of a fuzzy graph which are analogous to the concept double vertex graph and complete double vertex graph in crisp graph theory. We have studied the connected, effective and complete properties of these operations. Also we have obtained the degree of a vertex in the double vertex graph and complete double vertex graph of a fuzzy graph.

Keywords: Effective Fuzzy Graph, Connected Fuzzy Graph, Complete Fuzzy Graph, Double Vertex Graph of a Fuzzy Graph, Complete Double Vertex Graph of a Fuzzy Graph.

2010 Mathematics Subject Classification: 03E72, 05C07.

Introduction

Fuzzy graph theory was introduced by Azriel Rosenfeld in 1975. Later on, Bhattacharya^[1] gave some remarks on fuzzy graphs. Some operations on fuzzy graphs were introduced by Mordeson.J.N. and Peng.C.S.^[2] The conjunction of two fuzzy graphs was defined by Nagoor Gani.A and Radha.K.^[3] We defined the direct sum^[4], the strong product^[6], the lexicographic products^[8], maximal product^[9] of two fuzzy graphs and studied the properties of these operations. In this paper, we have introduced the concept of double vertex graph and complete double vertex graph of a fuzzy graph which are analogous to the concept double vertex graph and complete double vertex graph in crisp graph theory. We have studied the connected, effective and complete properties of these operations. Also we have obtained the degree of a vertex in the double vertex graph and complete double vertex graph of a fuzzy graph.

First let us recall some preliminary definitions that can be found in^{[1]-[11]}.

A fuzzy graph G is a pair of functions (σ, μ) where σ is a fuzzy subset of a non-empty set V and μ is a symmetric fuzzy relation on σ . The underlying crisp graph of $G:(\sigma, \mu)$ is denoted by $G^*:(V, E)$ where $E \subseteq V \times V$. A fuzzy graph $G:(\sigma, \mu)$ with underlying crisp graph $G^*:(V, E)$ is called a connected fuzzy graph if for all $u, v \in V$ there exists at least one non-zero path between u and v .

A fuzzy graph $G:(\sigma, \mu)$ is called an effective fuzzy graph if $\mu(uv) = \sigma(u) \wedge \sigma(v)$ for all $uv \in E$ and a complete fuzzy graph if $\mu(uv) = \sigma(u) \wedge \sigma(v)$ for all $u, v \in V$. Therefore G is complete if and only if G is effective and G^* is complete.

The degree of a vertex u of a fuzzy graph $G:(\sigma, \mu)$ with underlying crisp graph $G^*:(V, E)$ is defined as

$$d_G(u) = \sum_{u \neq v} \mu(uv) = \sum_{uv \in E} \mu(uv)$$

Let $G = (V, E)$ be a graph with order $n \geq 2$. The double vertex graph, denoted by $U_2(G)$, is the graph whose vertex set consists of all nC_2 unordered pairs of V such that two vertices $\{x, y\}$ and $\{u, v\}$ are adjacent if and only if $|\{x, y\} \cap \{u, v\}| = 1$ and if $x = u$, then y and v are adjacent in G .

Let $G = (V, E)$ be a graph with order $n \geq 2$. The complete double vertex graph, denoted by $CU_2(G)$, is the graph whose vertex set consists of all $(n+1)C_2$ unordered pairs of V , that is, it contains all the vertices of $U_2(G)$ and all 2-element multisets of the form $\{a, a\}$, such that two vertices $\{x, y\}$ and $\{u, v\}$ are adjacent if and only if $|\{x, y\} \cap \{u, v\}| = 1$ and if $x = u$, then y and v are adjacent in G . The following figure-1 illustrates the double vertex and the complete double vertex graphs of a graph.

Correspondence

S. Arumugam

B.T.Asst, Govt. Hr.Sec.
School, Kolakkudi,
Musiri Taluk, Tiruchirapalli-
621208

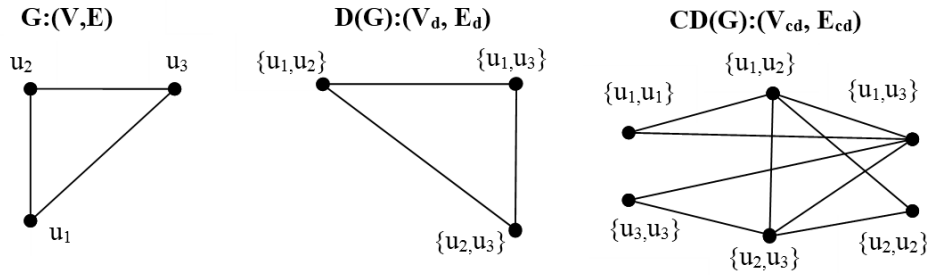


Figure-1

Notation

In a fuzzy graph $G:(\sigma, \mu)$ with underlying crisp graph $G^*:(V, E)$, the relation $\sigma \geq \mu$ means that $\sigma(u) \geq \mu(uv)$ for every $u \in V$ and for every $uv \in E$.

2. Double vertex graph of a fuzzy graph

2.1. Definition

Let $G:(\sigma, \mu)$ be a fuzzy graph with underlying crisp graph $G^*:(V, E)$ of order $n \geq 2$. Define $D(G):(\sigma_d, \mu_d)$ on $U_2(G^*):(V_d, E_d)$ whose vertex set V_d consists of all nC_2 unordered pairs of vertices in V such that two vertices $\{x, y\}$ and $\{u, v\}$ are adjacent if and only if $|\{x, y\} \cap \{u, v\}| = 1$ and if $x = u$ then y and v are adjacent in G such that,

$$\sigma_d(\{u_i, u_j\}) = \sigma(u_i) \wedge \sigma(u_j), \text{ for all } \{u_i, u_j\} \in V_d \text{ and}$$

$$\mu_d(\{u_1, u_2\} \{u_1, u_3\}) = \sigma(u_1) \wedge \mu(u_2 u_3), \forall \{u_1, u_2\} \{u_1, u_3\} \in E_d$$

$$\begin{aligned} \text{Now } \sigma(u_1) \wedge \mu(u_2 u_3) &\leq \sigma(u_1) \wedge [\sigma(u_2) \wedge \sigma(u_3)] \\ &= [\sigma(u_1) \wedge \sigma(u_2)] \wedge [\sigma(u_1) \wedge \sigma(u_3)] \\ &= \sigma(u_1, u_2) \wedge \sigma(u_1, u_3). \end{aligned}$$

Hence $\mu_d(\{u_1, u_2\} \{u_1, u_3\}) \leq \sigma(\{u_1, u_2\}) \wedge \sigma(\{u_1, u_3\})$. Therefore $D(G):(\sigma_d, \mu_d)$ is a fuzzy graph. This is called the double vertex graph of the fuzzy graph G .

2.2. Remark

If $G:(\sigma, \mu)$ has ‘n’ non zero vertices and ‘m’ non zero edges, then the double vertex graph $D(G):(\sigma_d, \mu_d)$ has $n(n - 1)/2$ non zero vertices and $m(n - 2)$ non zero edges.

2.3. Example

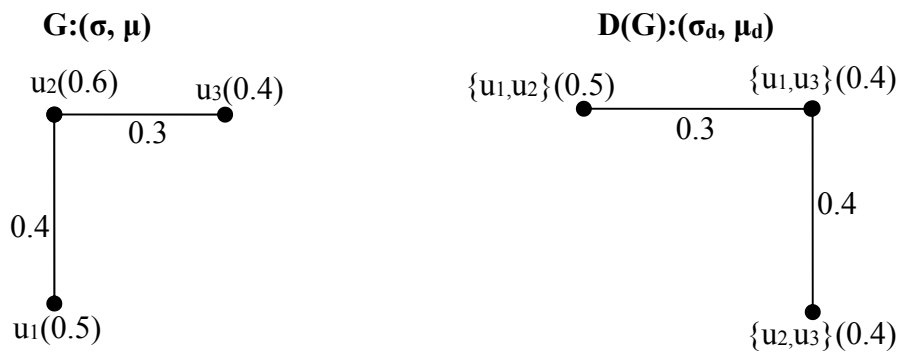


Figure-2

2.4. Theorem

The double vertex graph of an effective fuzzy graph is an effective fuzzy graph.

Proof:

Let $G:(\sigma, \mu)$ be an effective fuzzy graph. Then $\mu(u_1 u_2) = \sigma(u_1) \wedge \sigma(u_2)$ for all $u_1 u_2 \in E$.

Let $D(G):(\sigma_d, \mu_d)$ be the double vertex graph of G defined on $U_2(G^*):(V_d, E_d)$. Then proceeding as in the definition, $\sigma_d(\{u_i, u_j\}) = \sigma(u_i) \wedge \sigma(u_j)$, for all $\{u_i, u_j\} \in V_d$ and

$$\mu_d(\{u_1, u_2\} \{u_1, u_3\}) = \sigma(u_1) \wedge \mu(u_2 u_3), \forall \{u_1, u_2\} \{u_1, u_3\} \in E_d$$

$$\begin{aligned} \text{Now } \sigma(u_1) \wedge \mu(u_2 u_3) &= \sigma(u_1) \wedge [\sigma(u_2) \wedge \sigma(u_3)] = [\sigma(u_1) \wedge \sigma(u_2)] \wedge [\sigma(u_1) \wedge \sigma(u_3)] \\ &= \sigma(\{u_1, u_2\}) \wedge \sigma(\{u_1, u_3\}). \end{aligned}$$

Hence $\mu_d(\{u_1, u_2\} \{u_1, u_3\}) = \sigma(\{u_1, u_2\}) \wedge \sigma(\{u_1, u_3\})$. This is true for all $\{u_1, u_2\} \{u_1, u_3\} \in E_d$ and therefore $D(G):(\sigma_d, \mu_d)$ is an effective fuzzy graph.

2.5. Example

The following Figure-3 illustrates the theorem2.4.

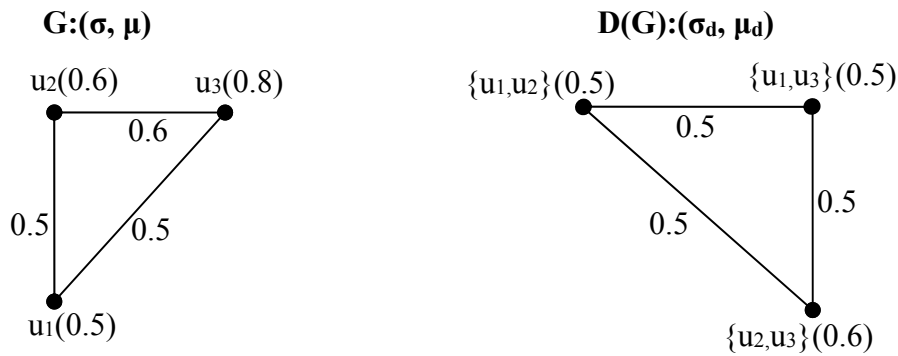


Figure-3

2.6. Theorem

The double vertex graph of a connected fuzzy graph is a connected fuzzy graph.

Proof

Let $G:(\sigma, \mu)$ be a connected fuzzy graph. Then $\mu^\infty(u_1, u_2) > 0$ for all $u_1, u_2 \in E$.

Let $D(G):(\sigma_d, \mu_d)$ be the double vertex graph of G defined on $U_2(G^*):(V_d, E_d)$.

To prove: $D(G)$ is connected.

Consider $V_i \subseteq V_d$ consisting all the vertices having u_i . That is, $V_i = \{\{u_i, u_1\}, \{u_i, u_2\}, \dots, \{u_i, u_m\}\}$ (say). Since each pair of vertices in V_i has u_i in common and G is connected, these

vertices form a connected component of G . Thus we have 'n' connected components of G for $i=1,2,\dots,n$ namely G_1, G_2, \dots, G_n . Now consider any two connected components G_1 and G_2 of G and $\{u_1, u_2\} \in G_1$. Suppose that u_1 is adjacent to u_k in G . Since G is connected, there exists a vertex $\{u_2, u_k\} \in G_2$ such that $\{u_1, u_2\}$ and $\{u_2, u_k\}$ are path connected. Thus $\mu_d^\infty(\{u, v\}, \{x, y\}) > 0$ for any $\{u, v\} \in G_1$ and $\{x, y\} \in G_2$. Thus the two components G_1 and G_2 are not disjoint. This is true for any two components and hence G is connected.

2.7. Example

The following Figure-4 illustrates the theorem2.5.

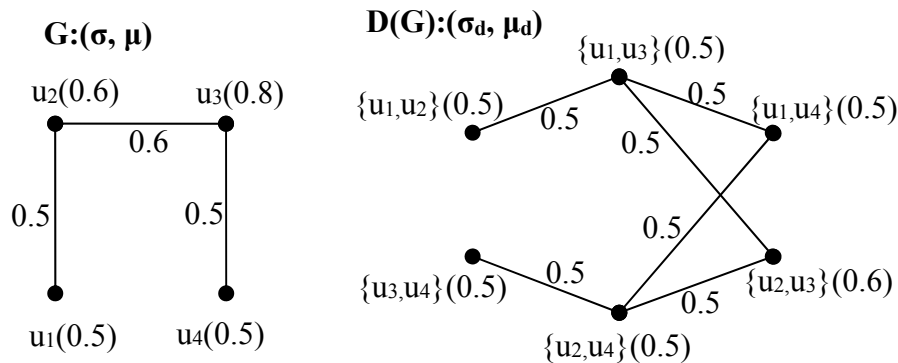


Figure-4

2.8. Remark

The double vertex graph of a complete fuzzy graph need not be a complete fuzzy graph. Consider the following fuzzy

graph $G:(\sigma, \mu)$ and its double vertex graph $D(G):(\sigma_d, \mu_d)$ where $G:(\sigma, \mu)$ is a complete fuzzy graph and $D(G):(\sigma_d, \mu_d)$ is not a complete one.

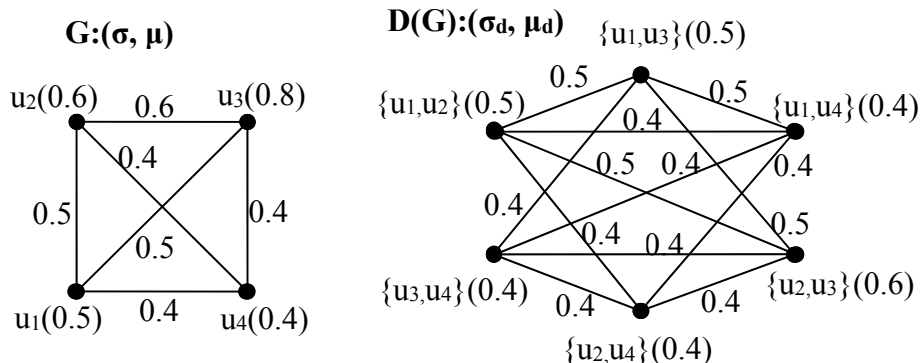


Figure-5

2.9. Remark

If $G:(\sigma,\mu)$ is a complete fuzzy graph with two vertices then double vertex graph $D(G):(\sigma_d, \mu_d)$ is a fuzzy graph with only one vertex.

If $G:(\sigma,\mu)$ is a complete fuzzy graph with three vertices then double vertex graph $D(G):(\sigma_d, \mu_d)$ is also a complete fuzzy graph with the same number of vertices and edges.

Since every complete fuzzy graph is an effective fuzzy graph, from theorem 2.4, the double vertex graph of a complete fuzzy graph is an effective fuzzy graph.

3. Degree of a vertex in the double vertex graph

The degree of any vertex in the double vertex graph of a fuzzy graph $G:(\sigma, \mu)$ is given by,

$$\begin{aligned} d_{D(G)}(\{u_i, u_j\}) &= \sum_{\{u_i, u_j\}, \{u_k, u_\ell\} \in E_d} \mu_d(\{u_i, u_j\}, \{u_k, u_\ell\}) \\ &= \sum_{u_i = u_k \text{ and } u_j, u_\ell \in E} \sigma(u_i) \wedge \mu(u_j, u_\ell) + \sum_{u_i, u_k \in E \text{ and } u_j = u_\ell} \sigma(u_j) \wedge \mu(u_i, u_k) \end{aligned}$$

3.1. Theorem

If $G:(\sigma,\mu)$ be a fuzzy graph such that $\sigma \geq \mu$, then the degree of a vertex in the double vertex graph $D(G):(\sigma_d, \mu_d)$ is given by,

$$d_{D(G)}(\{u_i, u_j\}) = d_G(u_j) + d_G(u_i) - 2\mu(u_i, u_j)$$

Proof:

Let $G:(\sigma,\mu)$ be a fuzzy graph such that $\sigma \geq \mu$. This implies that $\sigma \wedge \mu = \mu$. Then the degree of any vertex $\{u, v\} \in V_d$ is given by,

$$\begin{aligned} d_{D(G)}(\{u_i, u_j\}) &= \sum_{\{u_i, u_j\}, \{u_k, u_\ell\} \in E_d} \mu_d(\{u_i, u_j\}, \{u_k, u_\ell\}) \\ &= \sum_{u_i = u_k \text{ and } u_j, u_\ell \in E} \sigma(u_i) \wedge \mu(u_j, u_\ell) + \sum_{u_i, u_k \in E \text{ and } u_j = u_\ell} \sigma(u_j) \wedge \mu(u_i, u_k) \\ &= \sum_{u_i = u_k \text{ and } u_j, u_\ell \in E} \mu(u_j, u_\ell) + \sum_{u_i, u_k \in E \text{ and } u_j = u_\ell} \mu(u_i, u_k) \\ &= \sum_{u_i = u_k \text{ and } u_j, u_\ell \in E} \mu(u_j, u_\ell) + \mu(u_i, u_j) + \sum_{u_i, u_k \in E \text{ and } u_j = u_\ell} \mu(u_i, u_k) + \mu(u_i, u_j) - 2\mu(u_i, u_j) \\ &= d_G(u_j) + d_G(u_i) - 2\mu(u_i, u_j) \end{aligned}$$

3.2. Remark

In the above theorem 3.1 if $u_i, u_j \notin E$ then the degree of the vertex $\{u_i, u_j\}$ in the double vertex graph $D(G):(\sigma_d, \mu_d)$ is given by, $d_{D(G)}(\{u_i, u_j\}) = d_G(u_j) + d_G(u_i)$

3.3. Example

Consider the following fuzzy graph $G:(\sigma,\mu)$ in which $\sigma \geq \mu$ and its double vertex graph $D(G):(\sigma_d, \mu_d)$.

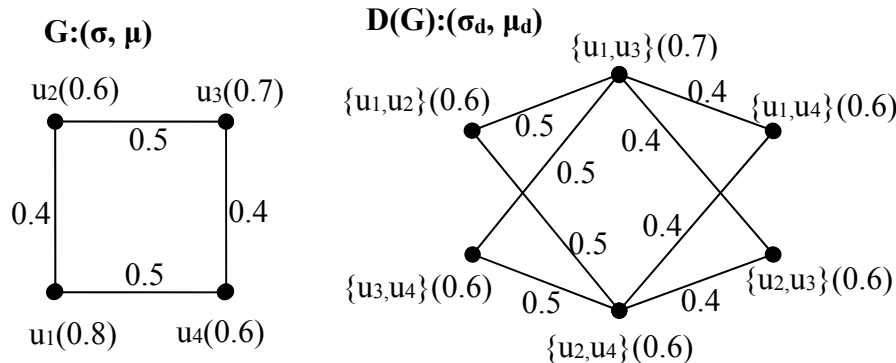


Figure-6

Now,

$$d_G(u_1) + d_G(u_2) - 2\mu(u_1, u_2) = 0.4 + 0.5 + 0.4 + 0.5 - 2(0.4) = 1.0 = d_{D(G)}(\{u_1, u_2\})$$

$$d_G(u_2) + d_G(u_3) - 2\mu(u_2, u_3) = 0.4 + 0.5 + 0.4 + 0.5 - 2(0.5) = 0.8 = d_{D(G)}(\{u_2, u_3\})$$

$$d_G(u_1) + d_G(u_3) = 0.4 + 0.5 + 0.4 + 0.5 = 1.8 = d_{D(G)}(\{u_1, u_3\}) \text{ since } u_1, u_3 \notin E$$

3.4. Remark

From the above example 3.3, it is clear that if a fuzzy graph $G: (\sigma, \mu)$ is regular then its double vertex graph $D(G): (\sigma_d, \mu_d)$ need not be regular.

4. Complete double vertex graph of a fuzzy graph

4.1. Definition

Let $G: (\sigma, \mu)$ be a fuzzy graph with underlying crisp graph $G^*: (V, E)$ of order $n \geq 2$. Define $D(G): (\sigma_{cd}, \mu_{cd})$ on $CU_2(G^*): (V_{cd}, E_{cd})$ whose vertex set V_{cd} consists of all $(n+1)C_2$ unordered pairs of vertices in V (duplicates allowed). That is, it contains all the vertices of $U_2(G)$ and all 2-element multi-sets of the form $\{a, a\}$. Again two vertices $\{x, y\}$ and $\{u, v\}$ are adjacent if and only if $|\{x, y\} \cap \{u, v\}| = 1$ and if $x = u$ then y and v are adjacent in G such that,

$$\sigma_{cd}(\{u_i, u_j\}) = \sigma(u_i) \wedge \sigma(u_j), \text{ for all } \{u_i, u_j\} \in V_{cd} \text{ and}$$

$$\mu_{cd}(\{u_1, u_2\} \{u_1, u_3\}) = \sigma(u_1) \wedge \mu(u_2 u_3), \forall \{u_1, u_2\} \{u_1, u_3\} \in E_{cd}$$

$$\begin{aligned} \text{Now } \sigma(u_1) \wedge \mu(u_2 u_3) &\leq \sigma(u_1) \wedge [\sigma(u_2) \wedge \sigma(u_3)] \\ &= [\sigma(u_1) \wedge \sigma(u_2)] \wedge [\sigma(u_1) \wedge \sigma(u_3)] \\ &= \sigma(u_1, u_2) \wedge \sigma(u_1, u_3). \end{aligned}$$

Hence $\mu_{cd}(\{u_1, u_2\} \{u_1, u_3\}) \leq \sigma(\{u_1, u_2\}) \wedge \sigma(\{u_1, u_3\})$. Therefore $CD(G): (\sigma_{cd}, \mu_{cd})$ is a fuzzy graph. This is called the complete double vertex graph of the fuzzy graph G .

4.2. Remark

If $G: (\sigma, \mu)$ has 'n' non zero vertices and 'm' non zero edges, then the complete double vertex graph $CD(G): (\sigma_d, \mu_d)$ has $n(n+1)/2$ non zero vertices and 'mn' non zero edges.

4.3. Example

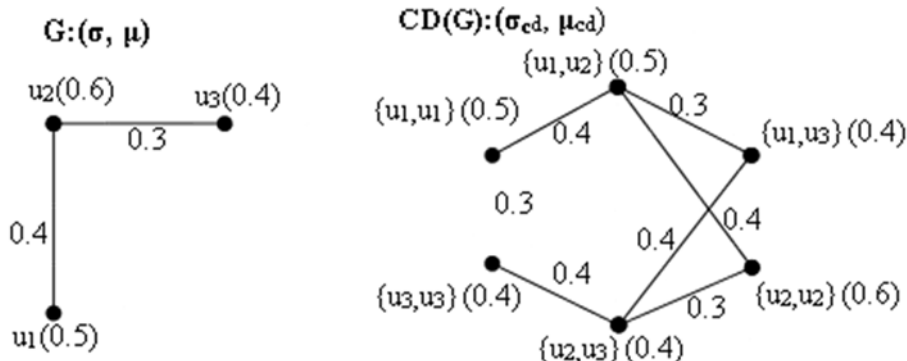


Figure-7

4.4. Theorem

The complete double vertex graph of an effective fuzzy graph is an effective fuzzy graph.

Proof:

Let $G: (\sigma, \mu)$ be an effective fuzzy graph. Then $\mu(u_1 u_2) = \sigma(u_1) \wedge \sigma(u_2)$ for all $u_1 u_2 \in E$.

Let $CD(G): (\sigma_{cd}, \mu_{cd})$ be the complete double vertex graph of G defined on $U_2(G^*): (V_{cd}, E_{cd})$. Then proceeding as in the definition, $\sigma_{cd}(\{u_i, u_j\}) = \sigma(u_i) \wedge \sigma(u_j)$, for all $\{u_i, u_j\} \in V_{cd}$ and

$$\mu_{cd}(\{u_1, u_2\} \{u_1, u_3\}) = \sigma(u_1) \wedge \mu(u_2 u_3), \forall \{u_1, u_2\} \{u_1, u_3\} \in E_{cd}$$

$$\begin{aligned} \text{Now } \sigma(u_1) \wedge \mu(u_2 u_3) &= \sigma(u_1) \wedge [\sigma(u_2) \wedge \sigma(u_3)] = [\sigma(u_1) \wedge \sigma(u_2)] \wedge [\sigma(u_1) \wedge \sigma(u_3)] \\ &= \sigma(\{u_1, u_2\}) \wedge \sigma(\{u_1, u_3\}). \end{aligned}$$

Hence $\mu_{cd}(\{u_1, u_2\} \{u_1, u_3\}) = \sigma(\{u_1, u_2\}) \wedge \sigma(\{u_1, u_3\})$. This is true for all $\{u_1, u_2\} \{u_1, u_3\} \in E_{cd}$ and therefore $CD(G): (\sigma_{cd}, \mu_{cd})$ is an effective fuzzy graph.

4.5. Example

The following Figure-8 illustrates the theorem 4.4.

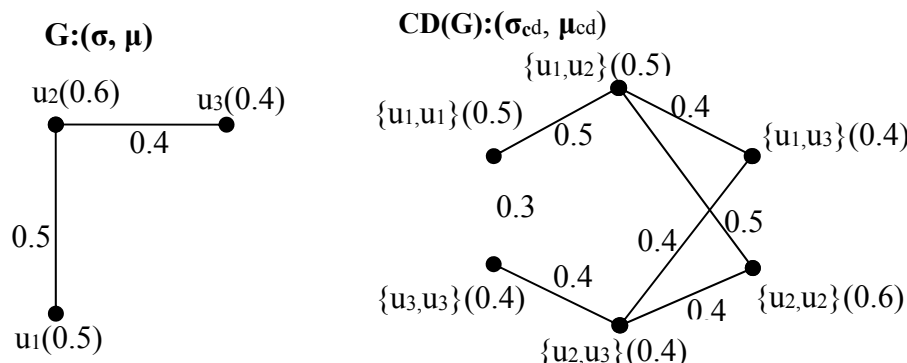


Figure-8

4.6. Remark

The complete double vertex graph of a complete fuzzy graph need not be a complete fuzzy graph. Consider the following

fuzzy graph $G: (\sigma, \mu)$ and its complete double vertex graph $CD(G): (\sigma_{cd}, \mu_{cd})$ where $G: (\sigma, \mu)$ is a complete fuzzy graph and $CD(G): (\sigma_{cd}, \mu_{cd})$ is not a complete fuzzy graph.

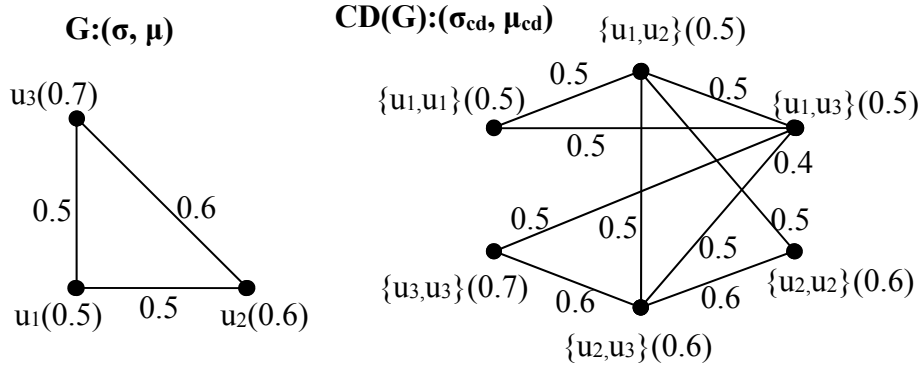


Figure-9

Since every complete fuzzy graph is an effective fuzzy graph, from theorem 4.4, the complete double vertex graph of a complete fuzzy graph is an effective fuzzy graph.

4.7. Theorem

The complete double vertex graph of a connected fuzzy graph is a connected fuzzy graph.

Proof: The proof is similar to that of 2.6.

$$d_{CD(G)}(\{u_i, u_j\}) = \sum_{\{u_i, u_j\} \{u_k, u_\ell\} \in E_{cd}} \mu_{cd}(\{u_i, u_j\} \{u_k, u_\ell\})$$

$$= \sum_{u_i = u_k \text{ and } u_j u_\ell \in E} \sigma(u_i) \wedge \mu(u_j u_\ell) + \sum_{u_i u_k \in E \text{ and } u_j = u_\ell} \sigma(u_j) \wedge \mu(u_i u_k)$$

Case (ii) if $i=j$,

$$d_{CD(G)}(\{u_i, u_i\}) = \sum_{\{u_i, u_i\} \{u_i, u_\ell\} \in E_{cd}} \mu_{cd}(\{u_i, u_i\} \{u_i, u_\ell\})$$

$$= \sum_{u_i u_\ell \in E} \sigma(u_i) \wedge \mu(u_i u_\ell)$$

5.1. Theorem

If $G:(\sigma, \mu)$ be a fuzzy graph such that $\sigma \geq \mu$, then the degree of a vertex in the complete double vertex graph $CD(G):(\sigma_{cd}, \mu_{cd})$ is given by,

$$d_{CD(G)}(\{u_i, u_j\}) = \begin{cases} d_G(u_j) + d_G(u_i), & \text{if } i \neq j \\ d_G(u_i) & \text{if } i = j \end{cases}$$

Proof:

Let $G:(\sigma, \mu)$ be a fuzzy graph such that $\sigma \geq \mu$. This implies that $\sigma \wedge \mu = \mu$. Then to find the degree of any vertex $\{u_i, u_j\} \in V_{cd}$, we have the following two cases.

Case (i) if $i \neq j$,

$$d_{CD(G)}(\{u_i, u_j\}) = \sum_{\{u_i, u_j\} \{u_k, u_\ell\} \in E_{cd}} \mu_{cd}(\{u_i, u_j\} \{u_k, u_\ell\})$$

$$= \sum_{u_i = u_k \text{ and } u_j u_\ell \in E} \sigma(u_i) \wedge \mu(u_j u_\ell) + \sum_{u_i u_k \in E \text{ and } u_j = u_\ell} \sigma(u_j) \wedge \mu(u_i u_k)$$

$$= \sum_{u_i = u_k \text{ and } u_j u_\ell \in E} \mu(u_j u_\ell) + \sum_{u_i u_k \in E \text{ and } u_j = u_\ell} \mu(u_i u_k)$$

$$= d_G(u_j) + d_G(u_i)$$

Case (ii) if $i=j$,

$$d_{CD(G)}(\{u_i, u_i\}) = \sum_{\{u_i, u_i\} \{u_i, u_\ell\} \in E_{cd}} \mu_{cd}(\{u_i, u_i\} \{u_i, u_\ell\})$$

$$= \sum_{u_i u_\ell \in E} \sigma(u_i) \wedge \mu(u_i u_\ell)$$

$$= \sum_{u_i u_\ell \in E} \mu(u_i u_\ell)$$

$$= d_G(u_i)$$

5.2. Example

Consider the fuzzy graph $G:(\sigma, \mu)$ and its complete double vertex graph $CD(G):(\sigma_{cd}, \mu_{cd})$ given below.

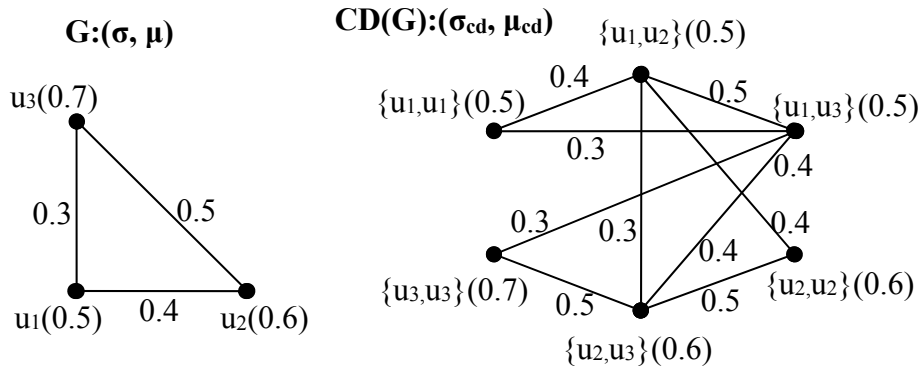


Figure-10

$$d_G(u_1) + d_G(u_2) = 0.4 + 0.3 + 0.4 + 0.5 = 1.6 = d_{CD(G)}(\{u_1, u_2\})$$

$$d_G(u_1) + d_G(u_3) = 0.4 + 0.3 + 0.3 + 0.5 = 1.5 = d_{CD(G)}(\{u_1, u_3\})$$

$$d_G(u_2) = 0.4 + 0.5 = 0.9 = d_{CD(G)}(\{u_2, u_2\})$$

$$d_G(u_1) = 0.4 + 0.3 = 0.7 = d_{CD(G)}(\{u_1, u_1\})$$

5.3. Remark

If a fuzzy graph $G:(\sigma, \mu)$ is regular then its complete double vertex graph $CD(G):(\sigma_{cd}, \mu_{cd})$ need not be regular.

5.4. Remark

Consider the fuzzy graph $G:(\sigma, \mu)$. Then, from the definitions of the double vertex graph $D(G):(\sigma_d, \mu_d)$ of G and the complete double vertex graph $CD(G):(\sigma_{cd}, \mu_{cd})$ of G , it is clear that $\sigma_d \subseteq \sigma_{cd}$ and $\mu_d \subseteq \mu_{cd}$. Hence $D(G):(\sigma_d, \mu_d)$ is a fuzzy sub graph of $CD(G):(\sigma_{cd}, \mu_{cd})$.

6. Conclusion

In this paper, we have introduced the concept of double vertex graph and complete double vertex graph of a fuzzy graph which are analogous to the concept double vertex graph in crisp graph theory. We have studied the connected, effective and complete properties of these two operations. Also we have obtained the degree of a vertex in the double vertex graph and complete double vertex graph of a fuzzy graph. In addition to the existing operations these operations and properties will also be helpful to study large fuzzy graph as a combination of small fuzzy graphs and to derive its properties from those of the small ones.

7. References

- 1 Bhattacharya. P, Some Remarks on Fuzzy Graphs, Pattern Recognition Letter 6 (1987), 297-302.
- 2 Mordeson J.N. and Peng C.S., Operations on fuzzy graphs, Information Sciences 79 (1994), 159-170.
- 3 Nagoorgani. A and Radha. K, Conjunction of Two Fuzzy Graphs, International Review of Fuzzy Mathematics, 2008, Vol. 3, 95-105.
- 4 Radha. K and Arumugam. S, On Direct Sum of Two Fuzzy Graphs, International Journal of Scientific and Research Publications, Volume 3, Issue 5, May 2013, ISSN 2250-3153.
- 5 Radha. K and Arumugam. S, Path Matrices of Fuzzy Graphs, Proceedings of the International Conference on Mathematical Methods and Computation, Jamal

- Academic Research Journal, Special Issue, February 2014, ISSN 0973-0303.
- 6 Radha. K and Arumugam. S, On Strong Product of Two Fuzzy Graphs, International Journal of Scientific and Research Publications, Volume 4, Issue 10, October 2014, ISSN 2250-3153.
- 7 Radha. K and Arumugam. S, Some Regular Properties of the Strong Product of Two Fuzzy Graphs, Global Journal for Research Analysis, Volume 3, Issue 11, November 2014, ISSN 2277-8160.
- 8 Radha. K and Arumugam. S, On Lexicographic Products of Two Fuzzy Graphs, International Journal of Fuzzy Mathematical Archive, Volume 7, No. 2, January 2015, ISSN 2320-3242 (P), 2320-3250 (online), 169-176.
- 9 Radha. K and Arumugam. S, On Maximal Product of Two Fuzzy Graphs, International Journal of Current Research, Volume 7, Issue 01, January 2015, ISSN 0975-833X, pp.11508-11515.
- 10 Radha. K and Arumugam. S, Regular Properties of Lexicographic Products, International Journal of Multidisciplinary Research and Development, Volume 2, Issue 3, March 2015, e-ISSN 2349-4182, p-ISSN 2349-5979,782-785.
- 11 Yousef Alavi, Mehdi Behzad, Paul Erdos, and Don R. Lick. Double Vertex Graphs. J. Combin. Inform. System Sci., 16(1):37-50, 1991.