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On special integer triples (m, n, k)

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Abstract

We search for non – zero positive integer values for m, n, k such that $m^2 - n^2 - k2mn$ is a perfect square. Considering m, n as the generators of $m^2 - n^2$, $m^2 + n^2$ and $2mn$, integer relations between the sides P, Q and special polygonal are exhibited.

Keywords: Integer triples, polygonal numbers and centered polygonal numbers
MSC Classification number: 11 D 99

1. Introduction

The Pythagorean triples offers an unlimited field for research. A set of positive integers $(a_1, a_2, a_3, \dots a_m)$ is said to have the property $D(n), n \in z - \{0\}$ if $a_i a_j + n$ is a perfect square for all $1 \le i \prec j \le m$ and such a set is called a Diophantine m-tuple with property $D(n)$. In [5] Diophantine triples for polygonal numbers from $t_{26,n}$ to $t_{35,n}$ and centered polygonal numbers from $ct_{26,n}$ to $ct_{35,n}$ in turn are presented. A sequence of strong rational Diophantine triples with the property $D\left((2n+1)^2 k^2\right)$, distinct rational quadruple (A, B, C, D) such that product of any two of them added with 4 is a perfect square are presented in [2]. In [1] the relations between the pairs of special polygonal numbers such that the difference in each pair is a perfect square. In this context one may refer [3, 4]. This communication concerns with yet another interesting triple (m,n,k) such that $m^2 - n^2 - k2mn$ is a perfect square. Also treating m, n as the generators of the Pythagorean triple T (P, Q, R) with $P = 2mn$, $Q = m^2 - n^2$, $R = m^2 + n^2$ relations between P, Q and special polygonal numbers are obtained.

Notations:

$t_{m,n} = n\left(1 + \frac{(n-1)(m-2)}{2}\right)$ = Polygonal number of rank n with sides' m

$ct_{m,n} = \frac{mn(n+1)}{2} + 1$ = Centered Polygonal number of rank n with sides m

$g_n = 2n-1$ = Gnomonic number

Method of Analysis

Let m, n and k be any non – zero positive integer values such that

$$m^2 - n^2 - k2mn = \alpha^2$$

Which is written as

$$(m - kn)^2 = (k^2+1)n^2 + \alpha^2 \quad (1)$$

We solve the above equation for the following four choices of α, namely

1) $\alpha = 8p + 4$
2) $\alpha = 4p - 1$
3) $\alpha = 10p + 4$
4) $\alpha = 6p - 4$

Choice 1: $\alpha = 8p + 4$

The integer values of m, n and p satisfying (1) are given by

$$m = 20q^3 + 12q^2 - 16q + 4$$
$$n = 16q^2 - 8q$$
$$p = \frac{4q^4 - 12q^2 + 16q - 8}{8}$$, an integer $q > 0$

Observation 1
Treating m, n as the generators of the Pythagorean triangle $T(P,Q,R)$ with $P = 2mn$, $Q = m^2 - n^2$, $R = m^2 + n^2$ it is seen that

$$Q - kP = 3ct_{24,p} + 2ct_{28,p} + 11$$

Choice 2: $\alpha = 4p - 1$
The integer values of m, n and p satisfying (1) are given by

$$m = 4q^4 + 8q^3 + 4q^2 - 4q + 1$$
$$n = 8q^2 - 4q \qquad p = q^4 + q$$

Observation 2
The Pythagorean triangle $T(P,Q,R)$ is such that

$$Q - kP = t_{22,p} + t_{10,p} + 4t_{3,p} + g_p + 2$$

Choice 3: $\alpha = 10p + 4$
For this choice 3, there are 3 sets of values of m,n and p which are given as follows

Set 1
$$m = 100q^4 - 100q^3 - 70q^2 - 60q + 36$$
$$n = 100q^2 - 60q$$
$$p = 10q^4 - 5q^3 - 5q^2 + 12q - 4$$

Set 2
$$m = 100q^4 - 100q^3 - 55q^2 - 40q + 16$$
$$n = 100q^2 - 40q$$
$$p = 10q^4 - 10q^3 - 5q^2 + 8q - 2$$

Set 3
$$m = 1000000q^4 - 339000q^3 + 52500q^2 - 3780q + 102$$
$$n = 100q^2 - 40q$$
$$p = 1000q^4 - 1600q^3 + 780q^2 - 122q + 9$$

Observation 3
Note that for each of the above sets, we have the relation

$$Q - kP = 9ct_{20,p} + 2t_{12,p} - g_p + 6$$

Choice 4: $\alpha = 6p - 4$
The integer values of m, n and p satisfying (1) are given by
$$m = 36q^4 + 72q^3 + 96q^2 + 24q + 4$$
$$n = 72q^2 + 24q$$
$$p = 6q^4 - 12q$$

Observation 4
The Pythagorean triangle $T(P,Q,R)$ is such that
$$Q - kP = ct_{24,p} + ct_{28,p} + ct_{20,p} - 42g_p - 29$$

2. Conclusion
To conclude that one may search for other triples under the equation considered.

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