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**P. Senthil Kumar**  
Assistant Professor,  
Department of Mathematics,  
Rajah Serfoji Government  
College (Autonomous),  
Thanjavur, Tamilnadu, India.

**A. Dinesh Kumar**  
Assistant Professor,  
Department of Mathematics,  
Dhanalakshmi Srinivasan  
Engineering College,  
Perambalur, Tamilnadu, India.

**M. Vasuki**  
Assistant Professor,  
Department of Mathematics,  
Srinivasan College of Arts and  
Science, Perambalur,  
tamilnadu, India.

## Mathematical model to find the gallbladder outcomes using normal distribution

**P. Senthil Kumar, A. Dinesh Kumar, M. Vasuki**

### Abstract

Estrogen therapy is thought to promote gallstone formation and cholecystitis but most data derive from observational studies rather than randomized trials. The aim of the study was to determine the effect of estrogen therapy in healthy postmenopausal women on gallbladder disease outcomes. The second order linear homogeneous differential equation was used to determine the effect of estrogen therapy in healthy postmenopausal women on gallbladder disease outcomes.

**Keywords:** Gallbladder, Estrogen Therapy, Normal Distribution.

**2010 Mathematics Subject Classification:** 60G12, 60H10

### 1. Introduction

Women were excluded if they had any illness that suggested less than a 3year's survival, had a prior cholecystectomy or gall bladder disease. Women with hysterectomy were eligible for the estrogen alone trial.

Eligible participants were randomized to 0.625 mg/d of conjugated equine estrogens (CEE) or placebo. Participants reported hospitalizations for gall bladder diseases and gall bladder related procedures, with events ascertained through medical record review. These data suggest an increase in risk of biliary tract disease among postmenopausal women using estrogen therapy [1] and [9].

In this paper, the model is characterized by the Markov Property of entering and exiting processes, by the service channel and by the system capacity to accommodate one customer at a time [4], [5], [6] and [7]. Here we use the function  $f_{T(1)}(t; 0)$  satisfies the second order

linear homogenous differential equation  $\frac{d^2}{dt^2} f_{T(1)}(t; 0) = -(2\lambda + \mu) \frac{d}{dt} f_{T(1)}(t; 0) - \lambda^2 f_{T(1)}(t; 0)$  with the initial conditions  $f_{T(1)}(0; 0) = 0$ ,  $\left. \frac{d}{dt} f_{T(1)}(t; 0) \right|_{t=0} = \lambda^2$ , the explicit value of  $f_{T(1)}(t; 0)$  is  $f_{T(1)}(t; 0) = \frac{\lambda^2}{\sqrt{\mu(4\lambda + \mu)}} e^{-(t/2)(2\lambda + \mu)} \left[ e^{(t/2)(\sqrt{\mu(4\lambda + \mu)})} - e^{-(t/2)(\sqrt{\mu(4\lambda + \mu)})} \right]$

### 2. Notations

|           |   |                 |
|-----------|---|-----------------|
| $\mu$     | - | Shape Parameter |
| $\lambda$ | - | Scale Parameter |
| $t$       | - | Time            |

### 3. Mathematical Model

Let mean value  $E(t) = E\{\cos d(P_0 P_t)\}$  satisfies

$$\frac{d^2}{dt^2} E = -\lambda \frac{d}{dt} E - c^2 E \quad (1)$$

With initial conditions  $\begin{cases} E(0) = 1 \\ \left. \frac{d}{dt} E(t) \right|_{t=0} = 0 \end{cases} \quad (2)$

**Correspondence:**  
**A. Dinesh Kumar**  
Assistant Professor,  
Department of Mathematics,  
Dhanalakshmi Srinivasan  
Engineering College,  
Perambalur, Tamilnadu, India.

And has the form (3)

$$E(t) = \begin{cases} e^{-\frac{\lambda t}{2}} \left[ \cosh \frac{t}{2} \sqrt{\lambda^2 - 4c^2} + \frac{\lambda}{\sqrt{\lambda^2 - 4c^2}} \sinh \frac{t}{2} \sqrt{\lambda^2 - 4c^2} \right] & 0 < 2c < \lambda \\ e^{-\frac{\lambda t}{2}} \left[ 1 + \frac{\lambda t}{2} \right] & \lambda = 2c > 0 \\ e^{-\frac{\lambda t}{2}} \left[ \cosh \frac{t}{2} \sqrt{4c^2 - \lambda^2} + \frac{\lambda}{\sqrt{4c^2 - \lambda^2}} \sinh \frac{t}{2} \sqrt{4c^2 - \lambda^2} \right] & 2c > \lambda > 0 \end{cases}$$

The solution to the problem (1) and (2) is given by

$$E(t) = \frac{e^{-\frac{\lambda t}{2}}}{2} \left[ \left( e^{\frac{t}{2} \sqrt{\lambda^2 - 4c^2}} + e^{-\frac{t}{2} \sqrt{\lambda^2 - 4c^2}} \right) + \frac{\lambda}{\sqrt{\lambda^2 - 4c^2}} \left( e^{\frac{t}{2} \sqrt{\lambda^2 - 4c^2}} - e^{-\frac{t}{2} \sqrt{\lambda^2 - 4c^2}} \right) \right] \tag{4}$$

So that (3) emerges [4] & [7]. For large values of  $\lambda$ , the first expression furnishes  $E(t) \sim 1$  and therefore the particle hardly leaves the starting point. If  $\frac{\lambda}{2} < c$ , the mean value exhibits an oscillating behavior; in particular, the

oscillations decrease as time goes on, and this means that the particle moves further and further reaching in the limit the poles of the sphere. In view of (4), we can prove the following.

In [7], The functions  $F_{n,1}^{(s)}(t) = \int_s^t dt_1 \dots \int_{t_{n-1}}^t dt_n e^{-\mu(t-t_n)} \prod_{i=1}^n [1 - e^{-\mu(t_i-t_{i-1})}]$

With  $t_0 = s$  do not depend on  $t$  but on the time interval  $[s, t]$ :

$$F_{n,1}^{(s)}(t) - F_{n,1}^{(0)}(t - s) \tag{5}$$

The functions  $F_{n,1}^{(0)}(t) = \int_0^t dt_1 \dots \int_{t_{n-1}}^t dt_n e^{-\mu(t-t_n)} \prod_{i=1}^n [1 - e^{-\mu(t_i-t_{i-1})}]$  satisfy the differential Equations

$$\frac{d^2}{dt^2} F_{n,1}^{(0)}(t) = -\mu \frac{d}{dt} F_{n,1}^{(0)}(t) + \mu F_{n-1,1}^{(0)}(t)$$

Where  $t_0 = 0, t > 0, n \geq 1$  (6)

The functions  $F_{n,0}^{(0)}(t) = \int_0^t dt_1 \dots \int_{t_{n-1}}^t dt_n e^{-\mu(t-t_n)} \prod_{i=2}^n [1 - e^{-\mu(t_i-t_{i-1})}]$  satisfy the differential equations

$$\frac{d^2}{dt^2} F_{n,0}^{(0)}(t) = -\mu \frac{d}{dt} F_{n,0}^{(0)}(t) + \mu F_{n-1,0}^{(0)}(t)$$

Where  $t > 0, n \geq 1, F_{n,0}^{(0)}(0) = 0$  (7)

In view of (7) we can prove the following,

The function  $f_{T(1)}(t; 0)$  satisfies the second order linear homogenous differential equation

$$\frac{d^2}{dt^2} f_{T(1)}(t; 0) = -(2\lambda + \mu) \frac{d}{dt} f_{T(1)}(t; 0) - \lambda^2 f_{T(1)}(t; 0)$$

With the initial conditions  $f_{T(1)}(0; 0) = 0, \frac{d}{dt} f_{T(1)}(t; 0) \Big|_{t=0} = \lambda^2$

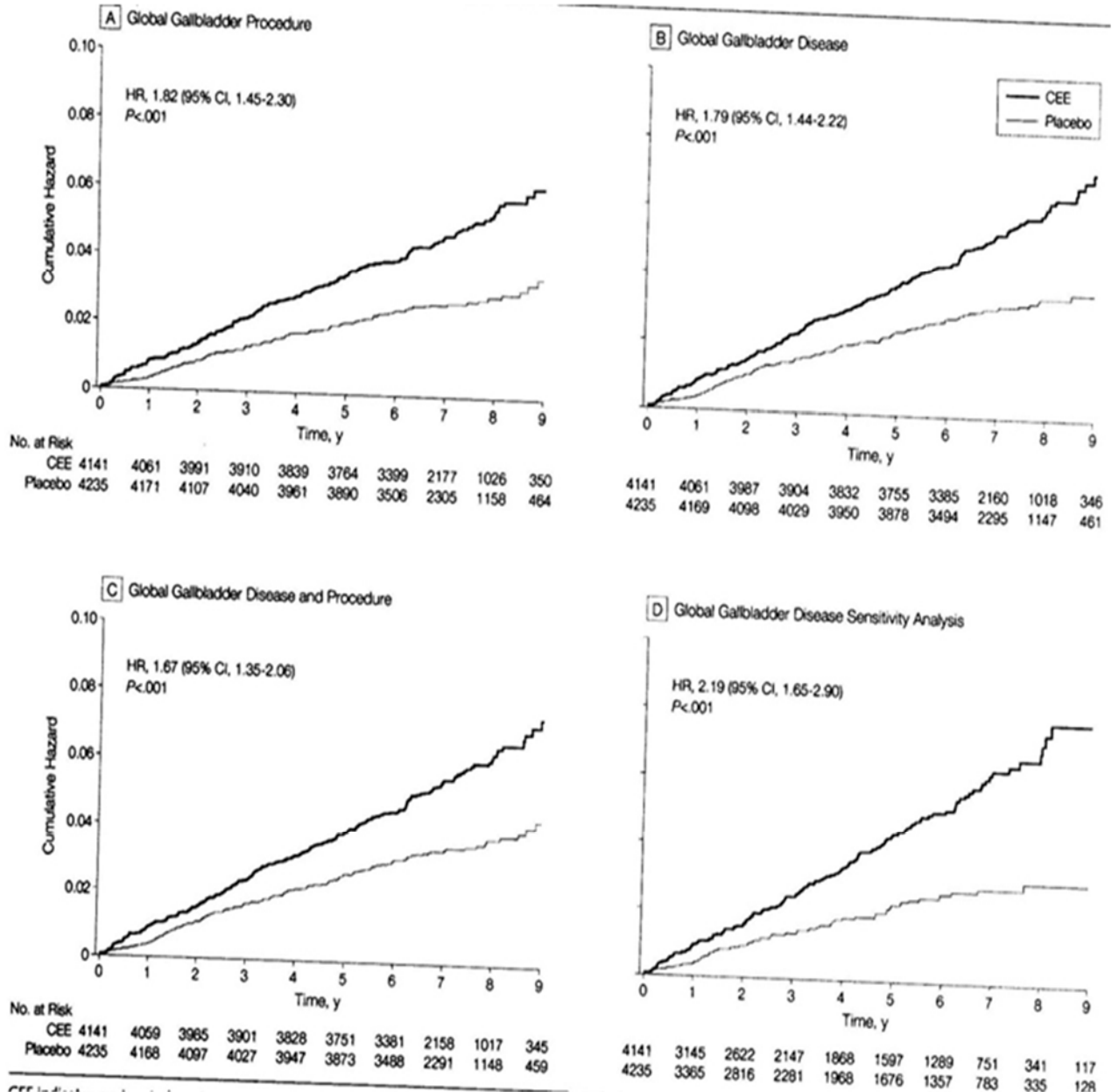
The explicit value of  $f_{T(1)}(t; 0)$  is

$$f_{T(1)}(t; 0) = \frac{\lambda^2}{\sqrt{\mu(4\lambda + \mu)}} e^{-(t/2)(2\lambda + \mu)} \left[ e^{(t/2)(\sqrt{\mu(4\lambda + \mu)})} - e^{-(t/2)(\sqrt{\mu(4\lambda + \mu)})} \right] \tag{8}$$

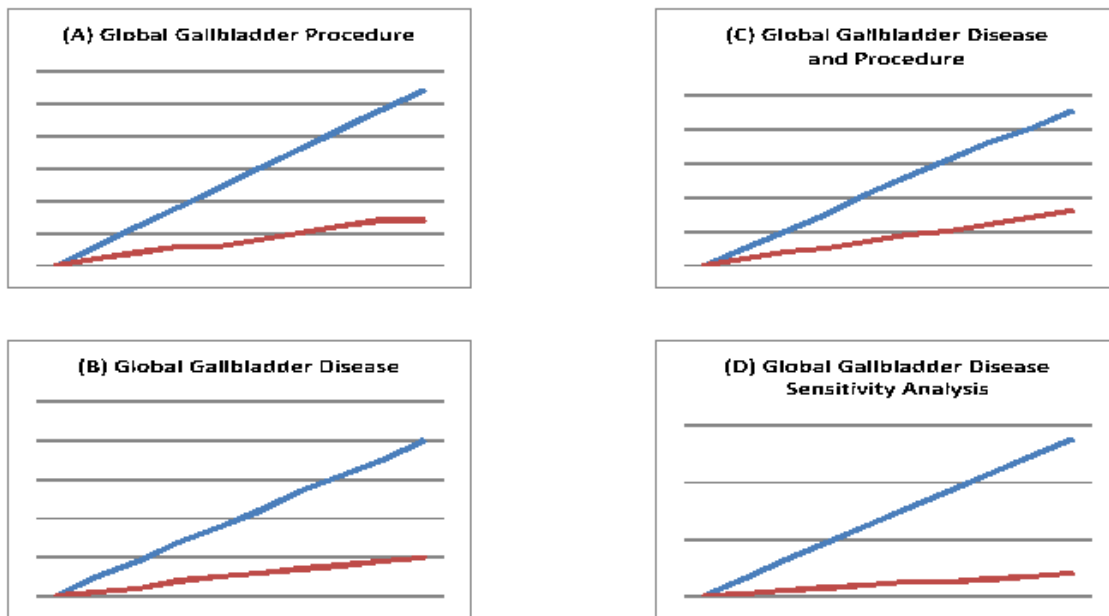
**4. Example**

Two randomized, double blind, placebo – controlled trials conducted at 40 US clinical centers. The volunteer sample was 22579 community – dwelling women aged 50 to 79years without prior cholecystectomy. Women with hysterectomy were randomized to 0.625mg/d conjugated equine estrogens (CEE) or placebo (n = 8376). The remaining 14203 are in the E+P arm. The proportion of women who were excluded from the analysis based on history of cholecystectomy was higher for the CEE trial compared with the E+P trial

[22% vs 14.5%,  $P < 0.001$ ] the Kaplan Meier estimates of cumulative hazards of any gallbladder procedure, any gallbladder disease event, and the global gallbladder disease or procedure measure showed a divergence starting in the first year randomization, with CEE separating earlier is shown in figure. After 6 months women's became nonadherent to randomization assignment in the sensitivity analysis, the resultant HR for the global gallbladder procedures and diseases outcome was 2.19 (nominal 95% CI, 1.65 – 2.90;  $P < 0.001$ ) for the CEE trial [2] [3] [8].



**Fig 1:** Kaplan Meier estimates of cumulative Hazards for any Gallbladder Outcomes in the Estrogen Alone Trial (Using Normal Distribution)



**Fig 2:** Kaplan Meier estimates of cumulative Hazards for any Gallbladder Outcomes in the Estrogen-Alone Trial (Using Normal Distribution)

### 5. Conclusion

The medical report suggests an increase in risk of biliary tract disease among postmenopausal women using estrogen therapy. The morbidity and cost associated with these outcomes may need to be considered in decisions regarding the use of estrogen therapy. The medical reports {Figure (1)} are beautifully fitted with the mathematical model {Figure (2)}; *i.e.* the results coincide with the mathematical and medical report.

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