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Matrix Representation and Data Compression for Structural Members under Bending Stress

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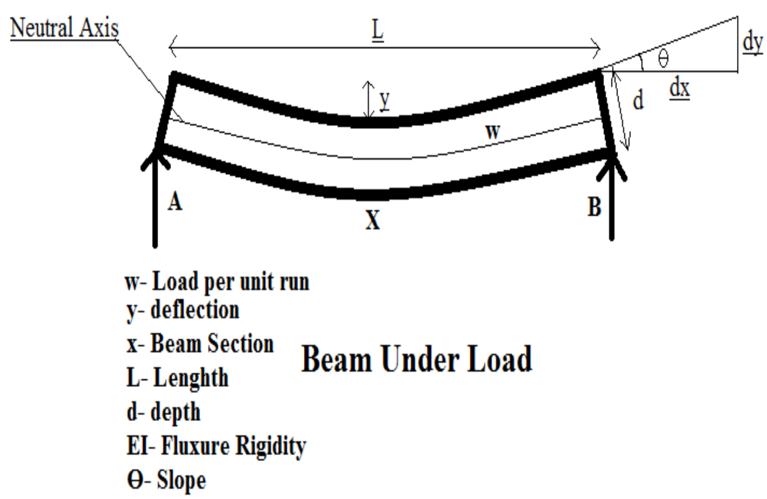
Abstract
Modern methods for solving problems of structural analysis uses matrix representation of various structural and dimensional properties of structural members. Currently adopted algorithms for solving these matrix is purely based on modern methods and few on classical methods. Proposed concept holds methods to use purely classical methods for new type of matrix representation of data in highly efficient and compressed form. Using classical equations like Macaulay's equation and methods of numerical mathematics, the whole information of a structural members needs to be stored in very low offset input values and saving the data base to evaluate future required values using interpolation. Proposed method uses the classical methods to compress the standard equations derived from Macaulay's Double Differential equation and represent it in compressed matrix form. Future software can be developed using these algorithms and input matrix for fast and efficient calculations.

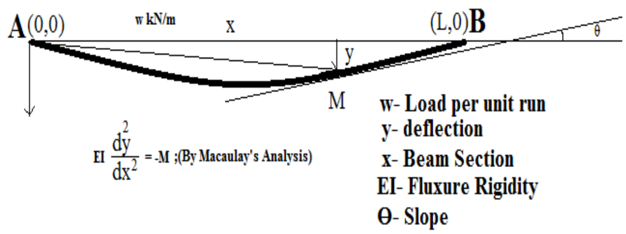
Keywords: Structural Analysis, Modern Methods, Macaulay's Equation, Bending of Beam

1. Introduction



The structural behavior of a member like beam simply supported at ends under self weight can be easily described by representing deflections and slope at each and every point throughout its span. Modern methods present to solve this problem includes Flexibility method and Rigidity method. The flexibility and rigidity matrix formation is a input method working on simple matrix algorithms with matrix addition and multiplication operations. These modern methods do not directly solve double differential equation or any integration operation. These matrix operation are limited in solving only deflections and slopes directly. Calculation of moments is done by usual classical methods like slope deflection method. There is a need to simplify these simple structural member analysis data using classical methods. Taking a case study of a beam under self weight and analyzing the data from it to create algorithms for modified method with data compression.

2. Case Study- Beam under Self Weight



A simply supported beam can be analyzed using Macaulay's Double differential equation. Moment at section x from A is directly dependent on the UDL or the self weight if the beam.

$$M = \frac{Wx^2}{2}$$

3. Solution to Above Condition

Substituting the value of moment into Macaulay's Equation and integrating w.r.t 'x' we get equation representing relations between deflection 'y' and slope at each point on beam.

$$EI\frac{dy}{dx} = -\frac{Wx^3}{6} + C_1$$

The above equation is a single order differential equation representing slopes for all points of a simply supported beam. End conditions for solving the unknown constant are y=0 at x=0 & x=L.

Further integrating the above differential equation w.r.t. x, we get the relation representing the deflection at every x distance from point A.

$$EIy = -\frac{Wx^4}{24} + C_1x + C_2$$

Putting end conditions and solving we get;

$$C_1 = \frac{WL^3}{24}$$

$$C_2 = 0$$

3.1 Slope

$$\frac{dy}{dx} = -\frac{Wx^3}{6EI} + \frac{WL^3}{24EI}$$

3.2 Deflection

$$y = -\frac{Wx^4}{24} + \frac{WL^3x}{24EI}$$

It can be generalized as for UDL;

$$y = A_1x^4 + A_2x + A3$$

4. Matrix Hypothesis

Generalizing the deflection equation and taking only constants into consideration, the deflection equation can be said to be purely dependent on constants A1, A2 & A3. Where these constants depends upon the end conditions and the beam. Let hypothesis a matrix containing these constants as its elements. For a beam it can be represented as;

4.1 Deflection Matrix

$$[Y] = \begin{bmatrix} A_1 & A_2 & A_3 \end{bmatrix}_{3\times1}$$

This 3*1 matrix holds all the information regarding the beam under bending. It can be used as an input for calculation of deflection at each and every point of the beam along its span. Proper algorithms can be formulized to interpolate the bending equation.

4.2 Slope Matrix

Using similar hypothesis Slope matrix can be generated as follows;

$$[\Theta] = \begin{bmatrix} B_1 & B_2 \end{bmatrix}_{2\times1}$$

This two matrix are the compressed form of the slope and deflection of the beam under load. The slope matrix is a function of deflection matrix. Only one input matrix i.e. the deflection is enough to generate the deflection and slope information of the case study object i.e. beam under gravity.

4.3 Slope Matrix Interpolation

$$[Y]^T \times \begin{bmatrix} 4 & 1 & 0 \end{bmatrix} = [\Theta]$$

Above algorithm shows the relationship between the deflection matrix and the slope matrix. With one input deflection matrix the slope matrix can be generated using above algorithm. Comparing the constants of the matrix of slope and deflection we get following relations and correlations;

$$A_1 = \frac{-W}{24EI}$$

$$A_2 = \frac{WL^3}{24EI}$$

$$A_3 = 0$$

$$B_1 = 4A_1 = -\frac{WL^3}{6EI}$$

$$B_2 = A_2 = \frac{WL^3}{24EI}$$

All the constant A1, A2, A3, B1 & B2 purely dependent upon the beam properties. Once deflection matrix formed, all the beam properties can be traced back to original values. The deflection matrix thus holds all the information needed to regenerate the analytical information of the beam under bending.

5. Regeneration Algorithm

5.1 Slope Equation

Using only the deflection matrix for all interpolations, the following sets of algorithms are derived for regeneration of solutions of Macaulay's double differential equation.

$$[\Theta] = [Y][4x^3 \ 1 \ 0]^T$$

$$= [A_1 \ A_2 \ A_3]\begin{bmatrix}4x^3\\1\\0\end{bmatrix}$$

$$= [4x^3A_1 \ A_2 \ 0]$$

$$= [B_1 \ B_2 \ 0]$$

$$\frac{dy}{dx} = \sum_{i=0}^{3} \Theta_i$$

5.2 Deflection Equation

The key variables constants are the integrated elements of the deflection matrix. An algorithm can be derived to extrapolate back the deflection equation i.e. the solution of the Macaulay's Double Differential Equation. The deflection matrix came out to be the compressed form of information that holds all the sufficient inputs for a software running on mentioned algorithms to regenerate the beam under bending.

$$[Y]_0 = [Y][x^4 \ x \ 1]^T$$

$$[Y]_0 = [A_1 \ A_2 \ A_3]\begin{bmatrix}x^4\\x\\1\end{bmatrix}$$

$$[Y]_0 = [A_1x^4 \ \ A_2x \ \ A_3]$$

$$y = \sum_{i=0}^{3} Y_i$$

$$y = A_1x^4 + A_2x + A_3$$

6. Acknowledgment

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7. References

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7. The sign on the left hand side of the equation depends on the convention that is used. For the rest of this article we will assume that the sign convention is such that a positive sign is appropriate.